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Light and Atomic Nucleus

("Pulse and Impulse of ELI" V)

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Common work with M Ganciu (equal contributions)

Forthcoming Seminar: M Ganciu, Laser-Heavy Ions Marriage
("Pulse and Impulse of ELI" VI)

1) High-Intensity Lasers (ELI program)

What we expect:

-Accelerated plasma electrons, protons

-Photonuclear reactions, giant dipole resonance

-Short-time atomic (chemical) processes (real time)

-Non-linear, multi-photon processes, HH X-ray generation

-Pair creation, including mesons

-Vacuum polarization, non-linear QED, light-by-light scatt

2) Acceleration of Heavy Ions (Nuclei)

-Nuclear structure and reactions information; -Higgs, Quark-Gluon Plasma (LHC, Geneva); -High-Energy Physics

3) Astrophysics

Nucleosynthesis, low-en reacts, exotic probes (rad-beams, laser)

Forward Beaming: -Stars are brighter or darker (aberration of light)

-Focusing or **collimating** a laser?

What is the Idea? Put all these 1)-3) together

What we have in mind

Ultra relativistic ion (bare nucleus), velocity v; $\varepsilon = 1TeV$ per nucleon
Photon beam ω_0 propagating counterwise, head-on collision

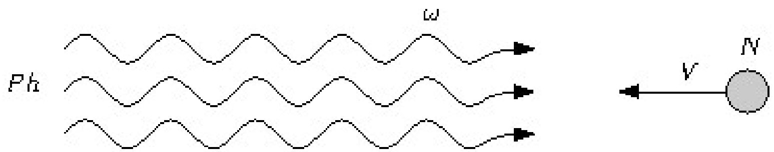
Doppler

$$\omega = \omega_0 \sqrt{\frac{1+\beta}{1-\beta}} \simeq 2\gamma\omega_0 \ , \ \beta = v/c \ , \ \beta \simeq 1 - \frac{\varepsilon_0^2}{2\varepsilon^2} \ , \ \omega \simeq 2\omega_0 \frac{\varepsilon}{\varepsilon_0} \ , \ \gamma = \varepsilon/\varepsilon_0$$

$$\varepsilon_0 \simeq 1GeV \ , \ \varepsilon = 1TeV \ , \ \gamma = 10^3 \ , \ \omega \simeq 2 \times 10^3 \omega_0$$

$1eV$-laser $\rightarrow 2keV$ $\quad$ $10keV$-(FE)laser $\rightarrow 20MeV$ (tunable)

We reach the nuclear energy scale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Typical optical laser

$\hbar\omega_0 = 1eV$ (wavelength $\lambda \simeq 1\mu m$), energy $\mathcal{E} = 50J$, pulse duration $\tau = 50fs$; Pulse length $l = 15\mu m$ (cca 15 wavelengths), power $P = 10^{15}w$ (1 pettawatt); Pulse area $d^2 = (15\mu m)^2$, intensity $I = P/d^2 = 4 \times 10^{20}w/cm^2$; Electric field $E \simeq 10^9 statvolt/cm$ ($1statvolt/cm = 3 \times 10^4 V/m$), magnetic field $H = 10^9 Gs$ ($1Ts = 10^4 Gs$) (high fields)

The relativistic ion sees a shortened pulse $l' = \sqrt{1-\beta^2}l$, shortened duration $\tau' = \sqrt{1-\beta^2}\tau$, energy $\mathcal{E}' = \mathcal{E}\sqrt{(1+\beta/(1-\beta)}$ (the number of photons $N_{ph} \simeq 10^{20}$ is invariant)

Electric field $E' = E/\sqrt{1-\beta} = \sqrt{2}(\varepsilon/\varepsilon_0)E \simeq 10^{12} statvolt/cm$: **two orders of magnitude below Schwinger limit**

Comment

Doppler $\omega = \omega_0 \sqrt{\frac{1+\beta}{1-\beta}} \simeq 2\gamma\omega_0$ $(\lambda' = \lambda/2\gamma)$

Sometimes, very often: $l' = \lambda' n' = \lambda' n = n/2\gamma$, $t' = t/2\gamma$, power (intensity) 2γ (from ω)x2γ (from t')$\Longrightarrow 4\gamma^2$! Wrong

Correct: $\omega \Longrightarrow 2\gamma$, $t' \Longrightarrow \gamma$ Power $P' = 2\gamma^2 P = 2 \times 10^{21} w$ (intensity $2\gamma^2 I = 8 \times 10^{26} w/cm^2$)

Collimation: Higher enhancement

Beam cross-section $D^2 = (0.5mm)^2$, Intensity $I = P/D^2 = 4 \times 10^{17} w/cm^2$, Electric field $E \simeq 5 \times 10^7 statvolt/cm$ (weaker)

In the rest frame beam area $D^{'2}$ decreases by a factor $(1-\beta)/(1+\beta) \simeq 1/4\gamma^2$ as a consequence of the "forward beaming" (aberration of light)

We have $D^{'2} = D^2(1-\beta)/(1+\beta)$, which leads to an enhancement factor $(1+\beta)/(1-\beta)^2$ for intensity and a factor $(1+\beta)^{1/2}/(1-\beta)$ $(\simeq 2\sqrt{2}\gamma^2)$ for field

We get, for instance, $I^{'} \simeq 3 \times 10^{24} w/cm^2$ and an electric field $E^{'} \simeq 2\sqrt{2}\gamma^2 E \simeq 10^{14} statvolt/cm$!!

Free Electron Laser

$\hbar\omega_0 = 10keV$, pulse duration $\tau = 50fs$, much lower energy $\mathcal{E} = 5 \times 10^{-5}J$ (power $P = 10Gw$)

Fields decrease by 3 orders of magnitude, but still they are very high: $10^9 - 10^{11} statvolt/cm$ in the rest frame of the accelerated ion

Therefore: nuclear energy scale, high fields, short-pulse duration

Basic Processes

Producing High-Energy Photons: 1) Bremsstrahlung, 2) Compton (3) Peripheral heavy ions collisions)

A) Incoming Photon upon a (free) charge (e, p): Relativistic or Non-Relativistic? Acceleration qE/m compared with "relativistic acceleration" $c\omega$ (qA vs mc^2)

For high ω nucleons are non-relativistic; electrons are!

B) Relativistic electrons: non-linear, multi-photon Compton, HH generation, photo-ionization, recollisions, light-by-light, pair creation (Breit-Wheeler)

Is the motion classical or quantum? What about radiation reaction?

C) High Fields: work $qE\lambda$ over Compton wavelength $\lambda = \hbar/mc$ compared with mc^2: Schwinger field $E = 10^{14} statvolt/cm$ (electrons) (10^{20} for protons): vacuum polarization, Non-linear QED, refractive index, light-by-light, pair production. What happens at high frequencies?

D) Pair production in the Coulomb field (Bethe-Heitler)

E) Direct photon-nucleus coupling

Pair Production in Coulomb Nuclear Field

Photons $\hbar\Omega = 10 MeV$; ultra-relativistic limit of the pair creation cross-section

Total cross-section

$$\sigma_{pair} = \frac{Z^2 r_0^2}{137}\left(\frac{28}{9}\ln\frac{2\hbar\Omega}{mc^2} - \frac{218}{27}\right) \simeq 10^{-28} Z^2 cm^2$$

class radius $r_0 = e^2/mc^2$; compare with nuclear cross-section $a^2 \simeq 10^{-26} cm^2$; $\sigma_{pair}/a^2 \simeq 10^{-2} Z^2$, may go as high as 10^2 for heavy nuclei (efficiency)

Direct Photon-Nucleus Coupling

Classical lagrangian $L = mv^2/2 - V + q\mathbf{v}\mathbf{A}/c - q\Phi$ (non-relat)

Momentum $\mathbf{p} = m\mathbf{v} + q\mathbf{A}/c$, hamiltonian

$$H = \frac{1}{2m}p^2 + V - \frac{q}{mc}\mathbf{p}\mathbf{A} + \frac{q^2}{2mc^2}A^2 + q\Phi$$

$p^2/2m + V$ is separated out, quantized

Remaining terms: perturbation, First order, external radiation $\mathbf{A} = \mathbf{A}_0$, $\Phi = 0$, $\mathbf{p} \simeq m\mathbf{v}$

Interaction hamiltonian

$$H_1 = -\frac{q}{c}\mathbf{v}\mathbf{A}_0 = -\frac{1}{c}\mathbf{J}\mathbf{A}_0$$

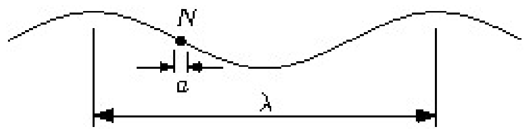
$\hbar\Omega = 10MeV \rightarrow \lambda \simeq 10^{-12}cm$, larger than nucleus $\simeq 10^{-13}cm$

Dipole approximation ($H_1 = -q\mathbf{r}\mathbf{E}_0 = -\mathbf{d}\mathbf{E}_0$)

$$H_1 = -\frac{1}{c}\sum_i \mathbf{J}_i\mathbf{A}_0$$

$$H_1(a,b) = -\tfrac{1}{c}\mathbf{J}(a,b)\mathbf{A}_0 =$$

$$= -\tfrac{1}{c}[\textstyle\sum_i \int d\mathbf{r}_1...d\mathbf{r}_i...d\mathbf{r}_N\psi_a^*(\mathbf{r}_1..\mathbf{r}_i..\mathbf{r}_N)\mathbf{J}_i\psi_b(\mathbf{r}_1..\mathbf{r}_i..\mathbf{r}_N)]\mathbf{A}_0 \ ,$$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Transition amplitude

$$c_{ab} = -\frac{i}{\hbar}\int dt H_1(a,b)e^{i\omega_{ab}t}\ ,\ \omega_{ab} = (E_a - E_b)/\hbar$$

$$\mathbf{A}_0(t) = \mathbf{A}_0 e^{-i\Omega t} + \mathbf{A}_0^* e^{i\Omega t}$$

Pulse duration $\tau' = \sqrt{1-\beta^2}\tau \simeq 5 \times 10^{-17}s$ much longer than the transition time $1/\Omega \simeq 10^{-22}s$

$$c_{ab} = \frac{2\pi i}{\hbar c}\mathbf{J}(a,b)\mathbf{A}_0\delta(\omega_{ab} - \Omega)$$

Number of transitions per unit time

$$P_{ab} = |c_{ab}|^2/t = 2\pi\left|\frac{\mathbf{J}(a,b)\mathbf{A}_0}{\hbar c}\right|^2\delta(\omega_{ab} - \Omega)$$

Criterion:

$J(a,b) \simeq qv/c$, $v/c = 10^{-1}$(for $\hbar\Omega = 10MeV$, rest energy $1GeV$)

Next, $\mathbf{E}_0 = (-1/c)\partial\mathbf{A}_0/\partial t = 10^9$, $\Omega = 10^{22}s^{-1} \rightarrow A_0 \simeq 10^{-3} statvolt$ (particle energy $qA_0 \simeq 1eV$)

$P_{ab} \simeq (10^{28}/\Delta\Omega)s^{-1}$, $\Delta\Omega \simeq 1/\tau' \simeq 10^{16}s^{-1}$ uncertainty in the pulse frequency

$P_{ab} \simeq 10^{12}s^{-1}$, much smaller than $\Omega = 10^{22}s^{-1}$

First-order calculations OK!

Two-Photon processes

Higher fields:

$$H_2 = -\frac{q^2}{2mc^2}\mathbf{A}_0^2$$

Dipole approx -No contribution (zero matrix elmts)

Still:

$$\mathbf{A}_0(\mathbf{r},t) = \mathbf{A}_0 e^{-i\Omega t + i\mathbf{k}\mathbf{r}} + \mathbf{A}_0^* e^{i\Omega t - i\mathbf{k}\mathbf{r}}$$

$$H_2(a,b) = -\frac{q^2}{2mc^2}\left[A_0^2(a,b)e^{-2i\Omega t} + A_0^{*2}(b,a)e^{2i\Omega t}\right]$$

$$A_0^2(a,b) = [\sum_i \int d\mathbf{r}_1 ... d\mathbf{r}_i ... d\mathbf{r}_N \psi_a^*(\mathbf{r}_1 .. \mathbf{r}_i .. \mathbf{r}_N) e^{2i\mathbf{k}\mathbf{r}_i} \psi_b(\mathbf{r}_1 .. \mathbf{r}_i .. \mathbf{r}_N)] A_0^2$$

Two-photon pocesses, **trans amplit**

$$c_{ab} = \frac{2\pi i}{\hbar}\frac{q^2}{2mc^2}A_0^2(a,b)\delta(\omega_{ab} - 2\Omega)$$

Comparing H_1 and H_2: qA_0/mc^2 (two-photons) *vs* v/c (one photon) Since $v/c \simeq 10^{-1}$, we should have $qA_0 > 10^{-1} \times 1GeV = 100MeV$ for two-photons; we have $qA_0 \simeq 1eV$!

Giant Dipole Resonance

Another process of excitation

$$H_{int} = -\frac{q}{mc}\mathbf{p}\mathbf{A} + \frac{q^2}{2mc^2}A^2 + q\Phi$$

or

$$H_{int} = -\frac{q}{c}\mathbf{v}\mathbf{A} - \frac{q^2}{2mc^2}A^2 + q\Phi$$

Mobile charges (protons) - displacement $\mathbf{u}(\mathbf{r}, t)$

Collective motion - particle-density

Center of mass of the charges (dipole approx)

Nuclear Polarization

Additional velocity $\dot{\mathbf{u}}$, charge $\rho_p = -nq div\mathbf{u}$ and current $\mathbf{j}_p = nq\dot{\mathbf{u}}$

Polarization, internal electromagnetic field, $\mathbf{A}_p$ and Φ_p

Retardation time $t_r = a/c \simeq 10^{-23}s$, $a \simeq 10^{-13}cm$, shorter than excitation time $\Omega^{-1} = 10^{-22}s$, the atomic nucleus **gets polarized**

$$H_{int} = H_1 - \frac{1}{c}\mathbf{J}\mathbf{A}_p - \frac{q}{c}\dot{\mathbf{u}}(\mathbf{A}_0 + \mathbf{A}_p) - \frac{q^2}{2mc^2}(\mathbf{A}_0 + \mathbf{A}_p)^2 + q\Phi_p$$

Dipole approx

$$H_{int} \simeq H_1 + H_{1p} \ , \ H_{1p} = -\frac{1}{c}\mathbf{J}\mathbf{A}_p$$

Dynamics for $\mathbf{u}$

$\mathbf{A}_p =$?

Internal forces, ω_c:

$$m\ddot{\mathbf{u}} = q(\mathbf{E}_0 + \mathbf{E}_p) - m\omega_c^2\mathbf{u}$$

Gauss's equation $div\mathbf{E}_p = 4\pi\rho_p = -4\pi nq div\mathbf{u}$, $\mathbf{E}_p = -4\pi nq\mathbf{u}$

(De-) polarizing factor f

$$\ddot{\mathbf{u}} + (\omega_c^2 + f\omega_p^2)\mathbf{u} = \frac{q}{m}\mathbf{E}_0$$

$\omega_p = \sqrt{4\pi nq^2/m}$ - plasma frequency

Numerical Estimations

Nucleons: $\hbar\omega_p \simeq Z^{1/2} MeV$ and $m\omega_c^2 d^2/2 = \mathcal{E}_c(d/a)$, d - displacement amplitude, $\mathcal{E}_c$ ($\simeq 7-8 MeV$) - mean cohesion energy per nucleon; $d = a/A^{1/3}$: $\hbar\omega_c \simeq 10 A^{1/6} MeV$

Effective frequency $\Omega_0 = (\omega_c^2 + f\omega_p^2)^{1/2} \simeq 10 MeV$

$$\ddot{\mathbf{u}} + \Omega_0^2 \mathbf{u} = \frac{q}{m}\mathbf{E}_0$$

Linear harmonic oscillator

$$\mathbf{E}_0 = \frac{i\Omega}{c}\mathbf{A}_0 e^{-i\Omega t} - \frac{i\Omega}{c}\mathbf{A}_0^* e^{i\Omega t}$$

Solution

Classical motion (or quantum?)

$$\mathbf{u} = -\frac{iq\Omega}{mc} \cdot \frac{1}{\Omega^2 - \Omega_0^2} \left(\mathbf{A}_0 e^{-i\Omega t} - \mathbf{A}_0^* e^{i\Omega t} \right)$$

$$\mathbf{E}_p = -4\pi f n q \mathbf{u} = \frac{if\omega_p^2 \Omega}{c} 2 \frac{1}{\Omega^2 - \Omega_0^2} \left(\mathbf{A}_0 e^{-i\Omega t} - \mathbf{A}_0^* e^{i\Omega t} \right)$$

$$\mathbf{A}_p = \frac{f\omega_p^2}{\Omega^2 - \Omega_0^2} (A_0 e^{-i\Omega t} + A_0^* e^{i\Omega t})$$

Nuclear Polarizability

Damping

$$\mathbf{u} = -\frac{q}{m}\mathbf{E}_0 \frac{1}{\Omega^2 - \Omega_0^2 + i\Omega\Gamma} e^{-i\Omega t} + c.c.$$

Polarization

$$\mathbf{P} = nqf\mathbf{u} = -\frac{f\omega_p^2}{4\pi} \frac{1}{\Omega^2 - \Omega_0^2 + i\Omega\Gamma} \mathbf{E}_0 e^{-i\Omega t} + c.c.$$

Polarizability

$$\alpha = -\frac{f\omega_p^2}{4\pi} \frac{1}{\Omega^2 - \Omega_0^2 + i\Omega\Gamma}$$

$$\mathbf{A}_p = -4\pi \left(\alpha \mathbf{A}_0 e^{-i\Omega t} + \alpha^* \mathbf{A}_0^* e^{i\Omega t}\right)$$

Transition amplitude

$$c_{ab} = -\frac{8\pi^2 i}{\hbar c}\alpha \mathbf{J}(a,b)\mathbf{A}_0 \delta(\omega_{ab} - \Omega)$$

Number of trans/time

$$P_{ab} = 32\pi^3 \left|\frac{\mathbf{J}(a,b)\mathbf{A}_0}{\hbar c}\right|^2 |\alpha|^2 \delta(\omega_{ab} - \Omega)$$

Rate of polarization-driven transitions - modified by

$$|\alpha|^2 = \left(\frac{f\omega_p^2}{4\pi}\right)^2 \frac{1}{(\Omega^2 - \Omega_0^2)^2 + \Omega^2\gamma^2}$$

Typical resonance factor, giant dipole resonance

Nuclear dielectric function, susceptibility

$$H_1 + H_{1p} = -\frac{1}{c}\mathbf{J}\left[(1-4\pi\alpha)\mathbf{A}_0 e^{-i\Omega t} + c.c.\right] = -\frac{1}{c}\mathbf{J}\left[\frac{1}{\varepsilon}\mathbf{A}_0 e^{-i\Omega t} + c.c\right]$$

$1 - 4\pi\alpha = (1+4\pi\chi)^{-1} = 1/\varepsilon$,

Interaction hamiltonian proportional to $1/\varepsilon = (\Omega^2 - \omega_c^2)/(\Omega^2 - \Omega_0^2)$

Similar for electrons

Plasma energy $\hbar\omega_p \simeq 10 Z^{1/2} eV$

Cohesion energy - Thomas-Fermi $16 Z^{7/3}/ZeV = 16 Z^{4/3} eV$, leads to $\hbar\omega_c \simeq 13 Z^{5/6} eV$

Scale energy $\hbar\Omega_0 \sim 1 keV$

Conclusions

Laser (optical, FEL) against (ultra-) relativistic (atomic) nuclei (heavy ions)

High (nuclear scale) energies, high field intensities

Photonuclear reactions (perturbation theory); giant dipole resonance

Nuclear polarizability, dielectric function

Forward beaming (collimation *vs* focusing)