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Gamma Laser Controlled by High External Fields

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Laser Dichotomy, **usually**: (two levels)

Narrow width for coherence, broader width for pumping

Optical Laser: third broader level, for pumping ($\sim 1eV$)

Nuclear laser: large energy ($10MeV$), Doppler effect, loss of coherence

Irrealizable! (yes or not?)

A further difficulty: coupling constant

To see the Difference Opt Laser vs Nuclear Laser we need a Theory

Laser Theory: **Does not exist!**

Discovery of the maser and the laser: 1950-1960... by engineers, physicists...

Townes, Maiman, Basov, Prokhorov, (Weber), ...

As regards the Theory, **Lamb**: We know everything and there exist

Three Schools of Thought:

Lamb&Scully, 2)Lax&Louisell, 3)Haken&Risken

Three Schools of Thought=No Theory!

The difficulty and the Failure of the Current "Theories"

Non-linear equations

Possible non-analyticity

Perturbation theory: fails

They "see" (predict) many things which do not exist

and do not see what does exist

What I mean by a Theory?

A simple problem:

Given: two quantum levels, interacting external and polarization fields (everything ideal)

Find: the population of the two levels, the population (intensity) of the fields as functions of time, preferrably stationary, coherence

A new concept:

Coherent coupling, all the atoms "excited" ("disexcited") in phase (stationary regime)

Direct coupling (3rd level not necessary), simple model

Sufficient condition: high external (pumping) field

Results: In principle, realizable, extremely low effficiency

Indeed non-analyticity

Practical idea (M Ganciu):

Relativistic electrons accelerated by intense laser pulses, Bremsstrahlung radiation, many photons, coupling with a 2 -level nuclear system

Usual problems with cross-section and Doppler scattering: in the coherent interaction context we may have surprises here (not discussed)

Still, another **difficulty**: coupling constant

Coherent interaction

Two levels $\hbar\omega_0 = \varepsilon_1 - \varepsilon_0$ (dipoles), mean inter-particle distance a, J_{01} matrix el particle current, interacting with a classical electromagnetic field

A coupling constant

$$\lambda = \frac{2g}{\hbar\omega_0} = \sqrt{\frac{2\pi}{3a^3\hbar\omega_0}}\frac{J_{01}}{\omega_0}$$

Critical condition

$$\lambda > 1$$

(at finite temperature $T < T_c$)

Second-order phase transition (super-radiance): macroscopic occupation of the two levels, macroscopic occup photon state, long-range order (of the quantum phases)

Typical atomic matter: $\lambda \sim 0.17$

Typical nuclear matter: $\lambda \sim 10^{-9}$ (this disparity makes the difference for the two lasers)

No chance for this transition

Mathematical Machinery: Fields

Vector potential (usual notations, transverse)

$$\mathbf{A}(\mathbf{r}) = \sum_{\alpha\mathbf{k}} \sqrt{\frac{2\pi\hbar c^2}{V\omega_k}} \left[\mathbf{e}_\alpha(\mathbf{k}) a_{\alpha\mathbf{k}} e^{i\mathbf{k}\mathbf{r}} + \mathbf{e}^*_\alpha(\mathbf{k}) a^*_{\alpha\mathbf{k}} e^{-i\mathbf{k}\mathbf{r}}\right]$$

Fields $\mathbf{E} = -(1/c)\partial\mathbf{A}/\partial t$, $\mathbf{H} = curl\mathbf{A}$

Three Maxwell's equations satisfied: $curl\mathbf{E} = -\frac{1}{c}\partial\mathbf{H}/\partial t$, $div\mathbf{H} = 0$, $div\mathbf{E} = 0$

Similar expression for the external vector potential $\mathbf{A}_0(\mathbf{r})$, the corresponding Fourier coefficients being denoted by $a^0_{\alpha\mathbf{k}}$, $a^{0*}_{\alpha\mathbf{k}}$

Classical lagrangian of radiation

$$L_f = \frac{1}{8\pi} \int d\mathbf{r} \left(E^2 - H^2 \right)$$

Interaction lagrangian

$$L_{int} = \frac{1}{c} \int d\mathbf{r} \cdot \mathbf{j} \left(\mathbf{A} + \mathbf{A}_0 \right) =$$

$$= \sum_{\alpha \mathbf{k}} \sqrt{\frac{2\pi\hbar}{\omega_k}} \left[\mathbf{e}_\alpha(\mathbf{k}) \mathbf{j}^*(\mathbf{k}) \left(a_{\alpha\mathbf{k}} + a^0_{\alpha\mathbf{k}} \right) + \mathbf{e}^*_\alpha(\mathbf{k}) \mathbf{j}(\mathbf{k}) \left(a^*_{\alpha\mathbf{k}} + a^{0*}_{\alpha\mathbf{k}} \right) \right]$$

Current density

$$\mathbf{j}(\mathbf{r}) = \frac{1}{\sqrt{V}} \sum_{\mathbf{k}} \mathbf{j}(\mathbf{k}) e^{i\mathbf{kr}}$$

(with $div \mathbf{j} = 0$, continuity equation)

Euler-Lagrange equations for the lagrangian $L_f + L_{int}$ lead to the wave equation with sources

$$\ddot{a}_{\alpha\mathbf{k}} + \ddot{a}^*_{-\alpha-\mathbf{k}} + \omega_k^2 \left(a_{\alpha\mathbf{k}} + a^*_{-\alpha-\mathbf{k}} \right) = \sqrt{\frac{8\pi\omega_k}{\hbar}} \mathbf{e}^*_\alpha(\mathbf{k})\mathbf{j}(\mathbf{k})$$

which is the fourth Maxwell's equation $curl\mathbf{H} = (1/c)\partial\mathbf{E}/\partial t + 4\pi\mathbf{j}/c$

Mathematical Machinery: Particles

N independent, non-relativistic, identical particles $i = 1, ...N$

Hamiltonian (internal degrees of freedom)

$$H_s = \sum_i H_s(i)$$

Orthonomal eigenfunctions $\varphi_n(i)$

$$H_s(i)\varphi_n(j) = \varepsilon_n \delta_{ij} \ , \ \int d\mathbf{r}\varphi_n^*(i)\varphi_m(j) = \delta_{ij}\delta_{nm}$$

Normalized eigenfunctions (for the whole ensemble)

$$\psi_n = \sum_i c_{ni}\varphi_n(i) = \frac{1}{\sqrt{N}} \sum_i e^{i\theta_{ni}}\varphi_n(i)$$

Field operator

$$\Psi = \sum_n b_n \psi_n$$

boson-like commutation relations $[b_n, b_m^*] = \delta_{nm}$, $[b_n, b_m] = 0$

Large, macroscopic values of the number of particles

$$N = \sum_n b_n^* b_n$$

The lagrangian

$$L_s = \frac{1}{2} \int d\mathbf{r} \left(\Psi^* \cdot i\hbar \partial \Psi / \partial t - i\hbar \partial \Psi^* / \partial t \cdot \Psi \right) - \int d\mathbf{r} \Psi^* H_s \Psi$$

or

$$L_s = \frac{1}{2} \sum_n i\hbar \left[b_n^* \dot{b}_n - \dot{b}_n^* b_n \right] - \sum_n \varepsilon_n b_n^* b_n$$

The hamiltonian

$$H_s = \sum_n \varepsilon_n b_n^* b_n$$

The corresponding equation of motion $i\hbar\dot{b}_n = \varepsilon_n b_n$ is Schrodinger's equation

It is worth noting that the same equation is obtained for b_n viewed as classical variables

Current density associated with this ensemble of particles

$$\mathbf{j}(\mathbf{r}) = \sum_i \mathbf{J}(i)\delta(\mathbf{r}-\mathbf{r}_i) = \frac{1}{V}\sum_{i\mathbf{k}} \mathbf{J}(i)e^{-i\mathbf{k}\mathbf{r}_i}e^{i\mathbf{k}\mathbf{r}} = \frac{1}{\sqrt{V}}\sum_{\mathbf{k}} \mathbf{j}(\mathbf{k})e^{i\mathbf{k}\mathbf{r}}$$

The interaction lagrangian

$$L_{int} = \sum_{nm\alpha\mathbf{k}} \sqrt{\frac{2\pi\hbar}{V\omega_k}} \left[\mathbf{e}_\alpha(\mathbf{k})\mathbf{I}^*_{mn}(\mathbf{k}) \left(a_{\alpha\mathbf{k}} + a^0_{\alpha\mathbf{k}}\right) + \mathbf{e}^*_\alpha(\mathbf{k})\mathbf{I}_{nm}(\mathbf{k}) \left(a^*_{\alpha\mathbf{k}} + a^{0*}_{\alpha\mathbf{k}}\right)\right] b^*_n b_m$$

where

$$\mathbf{I}_{nm}(\mathbf{k}) = \frac{1}{N} \sum_i \mathbf{J}_{nm}(i) e^{-i(\theta_{ni} - \theta_{mi})} e^{-i\mathbf{k}\mathbf{r}_i}$$

$\mathbf{J}_{nm}(i)$ are the matrix elements of the i-th particle current

Mathematical Machinery: Coherence

Interaction lagrangian re-written

$$L_{int} = \sum_{nm\alpha\mathbf{k}} \sqrt{\frac{2\pi\hbar}{V\omega_k}} F_{nm}(\alpha\mathbf{k}) \left(a_{\alpha\mathbf{k}} + a^*_{-\alpha-\mathbf{k}} \right) b^*_n b_m$$

$$F_{nm}(\alpha\mathbf{k}) = \frac{1}{N} \sum_i \mathbf{e}_\alpha(\mathbf{k}) \mathbf{J}_{nm}(i) e^{i\mathbf{k}\mathbf{r}_i - i(\theta_{ni} - \theta_{mi})}$$

First arrange a lattice of θ_{ni}

Reciprocal vectors $\mathbf{k}_r$, $r = 1, 2, 3$, $\hbar\omega_k = \varepsilon_n - \varepsilon_m > 0$

Arrange phases $\mathbf{k}_r \mathbf{r}_{pi} - (\theta_{ni} - \theta_{mi}) = K$

Then, L_{int} non-vanishing

Two levels: $n = 0$, $n = 1$

Macroscopic occupation, use c-numbers $\beta_{0,1}$ for operators $b_{0,1}$ (coherent states $b_{0,1}\left|\beta_{0,1}\right\rangle = \beta_{0,1}\left|\beta_{0,1}\right\rangle$)

Photon perators $a_{\alpha\mathbf{k}_r}$, $k_r = k_0$, $\hbar\omega_0 = ck_0$, replaced by c-numbers α

Interaction lagrangian

$$L_{int} = \sqrt{\frac{2\pi\hbar}{V\omega_0}} J_{01} \left[\left(\alpha + \alpha^0\right) + \left(\alpha^* + \alpha^{0*}\right)\right] \left(\beta_1^* \beta_0 + \beta_1 \beta_0^*\right)$$

The "classical" lagrangian

$$L_f = \frac{\hbar}{4\omega_0}\left(\dot{\alpha}^2 + \dot{\alpha}^{*2} + 2\left|\dot{\alpha}\right|^2\right) - \frac{\hbar\omega_0}{4}\left(\alpha^2 + \alpha^{*2} + 2\left|\alpha\right|^2\right)$$

$$L_s = \frac{1}{2}i\hbar\left(\beta_0^*\dot{\beta}_0 - \dot{\beta}_0^*\beta_0 + \beta_1^*\dot{\beta}_1 - \dot{\beta}_1^*\beta_1\right) - \left(\varepsilon_0\left|\beta_0\right|^2 + \varepsilon_1\left|\beta_1\right|^2\right)$$

$$L_{int} = \frac{g}{\sqrt{N}}\left[\left(\alpha + \alpha^0\right) + \left(\alpha^* + \alpha^{0*}\right)\right]\left(\beta_0\beta_1^* + \beta_1\beta_0^*\right)$$

Coupling constant

$$g = \sqrt{\pi\hbar/6a^3\omega_0}J_{01}$$

Equations of motion

$$\ddot{A} + \omega_0^2 A = \frac{2\omega_0 g}{\hbar\sqrt{N}} \left(\beta_0 \beta_1^* + \beta_1 \beta_0^*\right)$$

$$i\hbar\dot{\beta}_0 = \varepsilon_0 \beta_0 - \frac{g}{\sqrt{N}} \left(A + A^0\right) \beta_1$$

$$i\hbar\dot{\beta}_1 = \varepsilon_1 \beta_1 - \frac{g}{\sqrt{N}} \left(A + A^0\right) \beta_0$$

$A = \alpha + \alpha^*,\ A^0 = \alpha^0 + \alpha^{0*}$

Total hamiltonian

$$H_f^{tot} = \frac{\hbar}{4\omega_0}\left(\dot{A} + \dot{A^0}\right)^2 + \frac{\hbar\omega_0}{4}\left(A + A^0\right)^2$$

$$H_s = \varepsilon_0 \left|\beta_0\right|^2 + \varepsilon_1 \left|\beta_1\right|^2$$

$$H_{int} = -\frac{g}{\sqrt{N}}\left(A + A^0\right)\left(\beta_0\beta_1^* + \beta_1\beta_0^*\right)$$

Conserved, energy E,

$$H_f^{tot} + H_s + H_{int} = E$$

Number of particles, conserved

$$\left|\beta_0\right|^2 + \left|\beta_1\right|^2 = N$$

Stationary solutions $\beta_{0,1} = B_{0,1} e^{i\theta}$; equations of motion become

$$\ddot{A} + \omega_0^2 A = \frac{4\omega_0 g}{\hbar\sqrt{N}} B_0 B_1$$

$$i\hbar\dot{B}_0 - \hbar\dot{\theta} B_0 = \varepsilon_0 B_0 - \frac{g}{\sqrt{N}} \left(A + A^0\right) B_1$$

$$i\hbar\dot{B}_1 - \hbar\dot{\theta} B_1 = \varepsilon_1 B_1 - \frac{g}{\sqrt{N}} \left(A + A^0\right) B_0$$

The last two equations tell that $B_{0,1}$ and $\dot{\theta} = \Omega$ are constant

Particular solution of the first equation

$$A = \frac{4g}{\hbar\omega_0\sqrt{N}} B_0 B_1$$

In the absence of the external field ($A^0 = 0$) the solutions are given by

$$A = \frac{2g}{\hbar\omega_0}\sqrt{N}\left[1 - (\hbar\omega_0/2g)^4\right]^{1/2}$$

$$B_0^2 = \frac{1}{2}N\left[1 + (\hbar\omega_0/2g)^2\right]$$

$$B_1^2 = \frac{1}{2}N\left[1 - (\hbar\omega_0/2g)^2\right]$$

and frequency

$$\Omega = \omega_0\left[-\frac{1}{2} + \frac{2g^2}{\hbar^2\omega_0^2}\right]$$

where $\varepsilon_1 - \varepsilon_0 = \hbar\omega_0$ has been used and ε_0 was put equal to zero.

We can see: the ensemble of particles and the associated electromagnetic field can be put into a coherent state, the occupation amplitudes oscillating with frequency Ω, providing the critical condition

$$g > g_{cr} = \hbar\omega_0/2 \ , \ \lambda = 2g/\hbar\omega_0 > 1$$

The total energy of the coherence domain is given by

$$E = -\frac{g^2}{\hbar\omega_0} N \left[1 - (\hbar\omega_0/2g)^2\right]^2 = -\hbar\Omega B_1^2$$

It is lower than the non-interacting ground-state energy $N\varepsilon_0 = 0$

It may be viewed as the formation enthalpy of the coherence domains

This effect of seting up a coherence in matter is different from the lasing effect, precisely by this formation enthalpy

Rather, the picture emerging from the solution given here resembles to some extent a quantum phase transiton

The coupled ensemble of matter and radiation is unstable for a macroscopic occupation of the atomic quantum states and the associated photon states.

External field

Stationary solutions

$$A = 2\lambda\sqrt{N}\frac{\sqrt{\Omega(\Omega+1)}}{2\Omega+1}$$

$$B_0^2 = N\frac{\Omega+1}{2\Omega+1}\ ,\ B_1^2 = N\frac{\Omega}{2\Omega+1}$$

$\lambda = 2g/\hbar\omega_0$

Ω (measured in ω_0) given by

$$\Omega(\Omega+1) = \frac{\lambda^2}{4N}\left(\frac{2\Omega+1}{2\Omega+1-\lambda^2}\right)^2 A^{02}$$

Check that these solutions coincide formally with the solutions for zero external field), except for Ω ($\Omega > 0$) being given by $2\Omega+1-\lambda^2 = 0$ (the pole)

Dispersion equation above has always a unique solution $\Omega > 0$, which shows that the **coherent state is possible** and can be set up under the action of the external field. Since $\lambda \ll 1$ however, the effect is small for weak external fields.

Assume the external field high enough, such as parameter $x = \lambda A^0/\sqrt{N}$ is finite Take advantage of $\lambda \ll 1$ and simplify the above equations (leading contributions in λ)

Get the frequency

$$\Omega = \frac{1}{2}\left(\sqrt{x^2+1} - 1\right)$$

and

$$A = \frac{\lambda^2}{\sqrt{x^2+1}} A^0 = \lambda\sqrt{N}\frac{x}{\sqrt{x^2+1}}$$

$$B_0^2 = \frac{1}{2}N\left(1 + \frac{1}{\sqrt{x^2+1}}\right) \ , \ B_1^2 = \frac{1}{2}N\left(1 - \frac{1}{\sqrt{x^2+1}}\right)$$

These solutions coincide with the solutions for zero external field provided we make the formal change $\lambda^2 \to \sqrt{x^2+1}\,(>1)$

See that the polarization field A is much weaker than the external field A^0 (since $\lambda \ll 1$)

Total energy (leading contributions in λ)

$$H_f^{tot} = \frac{1}{4}A^{02} + N\frac{x^2}{2\sqrt{x^2+1}}$$

$$H_s = \frac{1}{2}N\left(1 - \frac{1}{\sqrt{x^2+1}}\right) \ , \ H_{int} = -N\frac{x^2}{2\sqrt{x^2+1}}$$

See that the increase in the field energy due to the polarization field is canceled out by the interaction energy (H_{int}), allowing thus to **pump energy** in the upper level (H_s) by an external field

The discharge of the energy H_s is a **lasing effect**

Field energy $H_f = \frac{1}{4}\hbar\omega_0 A^{02}$

Lasing energy $H_s = \frac{1}{4}N\hbar\omega_0 x^2 = \lambda^2 H_f$!!! (small λ)

This **makes the difference**: $\lambda = 10^{-9}$ for gamma, $\lambda = 0.1$ for optical lasers

Discussion&Conclusions

Assume the total bremsstrahlung energy radiated by one electron δE

Out of it, only the fraction corresponding to $\hbar\omega_0$ is effective in the process considered here

Denote by f this fraction

It can be estimated (roughly) by

$$f = \frac{I(\omega_0)}{\int d\omega I(\omega)} \Delta\omega_0$$

where $I(\omega)$ is the intensity of the bremsstrahlung radiation and $\Delta\omega_0$ is the spread in frequency of the level $\hbar\omega_0$

Rough estimation $f = \Delta\omega_0/\Delta\omega$, where $\Delta\omega$ is a reasonable frequency range of the bremsstrahlung radiation

Get an estimate for A^0 by

$$f\delta E\delta N = \frac{1}{4}\hbar\omega_0 A^{02}$$

where δN is the number of electrons in the pulse

Previous estimations: a laser pulse with wavelength 1μ, intensity $10^{18} w/cm^2$ and size $r = 1mm$, may accelerate relativistic electrons in a rarefied plasma with a group velocity close to the velocity of light (energy $\simeq 17MeV$ for instance, for a sample with $10^{18} cm^{-3}$ plasma electrons)

The number of these electrons is of the order of $\delta N = 10^{11}$ per pulse

Take, as a rough approximation, $\Delta\omega_0 = 10keV$ and $\Delta\omega = 100MeV$, and get $f = 10^{-4}$

Estimate the energy δE as the Coulombian energy of a nucleus with charge Z at distance of the order of a: $\delta E = Ze^2/a \simeq 10^3 eV$

Get $A^0 \simeq 60$ for $\hbar\omega_0 = 10MeV$

For a spot of linear size $r = 1mm$ the number N of nuclei can be taken approximately $N \simeq 10^{19}$

So we have $x = \lambda A^0/\sqrt{N} \simeq 10^{-18}$ for $\lambda = 10^{-9}$

This is a very small value for the parameter x, which indicates an **extremely poor efficiency of the process**

Total field energy per spot is of the order of $10^{10} eV$

It corresponds to cca $A^{02} \simeq 10^3$photons of energy $10 MeV$

Total lasing energy $\sim \lambda^2 \times 10^{10} eV \simeq 10^{-8} eV$!!! $(H_s = \lambda^2 H_f)$

No hope

Recall

$$f\delta E\delta N = \frac{1}{4}\hbar\omega_0 A^{02}$$

Recall

$$\delta N = n_p r^2 \lambda_l \frac{\omega_p^2}{4mc^2\omega_l^2}\sqrt{\pi\varepsilon_{el}W_0}$$

Use it for $x = \lambda A^0/\sqrt{N}$: $x^2 \simeq 10^{-43}\sqrt{\frac{W_0}{r^3}}$ (10^{-36})

Increase $W_0 = 10kJ$ by 2 orders; decrease $r = 1mm$ by 2 orders; gain 4 orders **Totally Insufficient**!!!

Other comments

Frequency spread $\Delta\omega_0$ related to the lifetime of the upper level, $\tau \sim \hbar/\Delta\omega_0$

For $\Delta\omega_0 = 10keV$ we get $\tau \sim 10^{-19}s$, which is very small in comparison with the laser pulse duration $\sim 10^{-12}s$

Would be desirable to have a more sharper energy level, which reduces further the efficiency of the process

Technical evaluation of the experimental implementation of such a process there are many other points to be assessed, like, for instance, the cross-section of the nuclear photoreaction, the Doppler effect, the consequences of a multi-level nuclear model, etc

In the context of a coherent interaction such questions may acquire different aspects than the usual ones

Though hopeless, such points might still be left for a forthcoming investigation

In conclusion, we may say that a coherent interaction of a two-level nuclear system with a high-intensity radiation field may lead, in principle, to a lasing effect, controlled by the external field, though with an extremely low efficiency

A technical point

Recall equation

$$i\hbar\dot{\beta}_1 = \hbar\omega_0\beta_1 - \frac{g}{\sqrt{N}}\left(A + A^0\right)\beta_0$$

Neglect here A; Schrodinger equation for the amplitude of the excitation rate

Compute it to the $1st$ order of the perturbation theory (standard)

$$|\beta|^2 = \left(\frac{2gA^0}{\hbar\sqrt{N}}\right)^2 \frac{\sin^2(\Delta\omega_0 t/2)}{(\Delta\omega_0)^2} = 2\pi t\left(\frac{gA^0}{\hbar\sqrt{N}}\right)^2 \delta(\Delta\omega_0)$$

where $\Delta\omega_0 = \omega - \omega_0$

The rate of excitation

$$w = |\beta|^2 / t = \frac{\pi\omega_0^2}{2} x^2 \delta(\Delta\omega_0)$$

multiplied with the number of states $\Delta\nu = 2V(4\pi k_0^2 \Delta k_0)/(2\pi)^3$ gives $w\Delta\nu = 2r^3\omega_0^4 x^2/3c^3$(for $V = 4\pi r^3/3$) and an excitation yield per pulse

$$|\beta|^2 = w\Delta\nu r/c = \frac{2}{3} (\omega_0 r/c)^4 x^2$$

This is to be compared with the yield in the stationary regime $B_1^2/N = x^2/4$

$$|\beta|^2 / N \simeq 10^{23} x^2$$

The rate of disexcitation processes!!! (Beware the perturb calcls!)

It is worth interesting another aspect

Making use of $x = 10^{-18}$ we get an excitation yield $|\beta|^2 = 10^6$ in the time $\tau = r/c \simeq 10^{-12}s$, *i.e.* $|\beta|^2/\tau \simeq 10^{17}$ excitation processes per second (and a similar figure for the number of disexcitation proecsses)

This means that a given nucleus undegoes $10^{17}/N \simeq 10^{-2}$ excitation processes per second

Similar process for an optical laser: $\hbar\omega_0 = 1eV$, energy $W_0 = 10^{23}eV$ (per spot), coupled directly to a two-level atomic system with the same energy $\hbar\omega_0 = 1eV$

Field energy $W_0 = \hbar\omega_0 A^{02}/4$ gives much more photons, $A^0 \simeq 10^{11}$

Lasing energy $H_s = \lambda^2 W_0 \simeq 10^{22}eV$ ($\simeq 1J$), for $\lambda \simeq 0.5$ (for $\hbar\omega_0 = 1eV$)

(actually much more!)

This is a much higher energy than for the nuclear system, as expected

It corresponds to $x \simeq 10$, which shows indeed that the pumping is more efficient

Similarly, the excitation yield ($|\beta|^2$) is $\simeq 10^{16}$, *i.e.* 10^{28} excitation processes per second, and 10^9 such processes for a given atomic particle

This is a much more efficient process that the corresponding process for a nuclear system

The main reason for this disparity resides in the difference between the coupling constants λ.

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