STATISTICAL PHYSICS FOR ANOMALOUS TRANSPORT IN PLASMAS

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1. Transport regimes in turbulent plasmas; the decorrelation trajectory method.

The approach developed in the frame of this collaboration for studying transport in stochastic fields, the decorrelation trajectory method, was used for the study of the transport coefficients in several conditions. The method itself was better analysed and the conditions of applicability were better understood. It is the only method that can describe the trajectory trapping specific to the ExB stochastic drift motion in turbulent tokamak plasmas.

As we have shown, the presence of trapping (at large Kubo numbers) determines anomalous diffusion regimes for test particles. As the parallel motion has an essential role in such self-consistent studies of turbulence, we have developed our method to include the electrostatic force along the magnetic field lines determined by the stochastic potential. We have determined the influence of the parallel motion on the diffusion coefficient and we have shown that this motion is important for electrons and produces a second kind of trajectory trapping. We have also determined the transport coefficients of collisional particles in stochastic magnetic fields. The process of magnetic line trapping was taken into account and we have shown that it determines anomalous diffusion regimes. A complex computer code, which calculates the diffusion coefficient for given parameters of the plasma collisions and of the stochastic magnetic fields, was developed. A long series of computer runs, which determine the diffusion regimes for test particles in magnetic turbulence, was performed.

The accuracy of the decorrelation trajectory method was evaluated by applying this method to several particular diffusion problems which have known analytical results.

The development of the decorrelation trajectory method that will permit the study of instabilities and relative transport was initiated. The first step consists in determining the correlations of the potential and trajectory fluctuations in subensembles of realisations. We have derived a close system of equation for the average trajectory and dispersion in subensembles. We have written and tested a code, which solves this system, and we have obtained preliminary results.

The results we have obtained in these studies are published in the following references: [1], [5], [8], [10], [12], [13], [16] and [17].

We present here shortly some of the main results and conclusions.

Electrostatic turbulence

We have shown that particle diffusion in electrostatic turbulence can strongly be influenced by the motion along the confining magnetic field if the parallel correlation length is finite [1]. The strongest effects appear in the radial diffusion of the fast "passing" electrons which have a free motion along the magnetic lines. The parallel motion determines a decrease of the diffusion coefficient D when the turbulence is weak (K<1) and an increase of D for a low frequency or high amplitude turbulence (K>1). An anomalous diffusion regime in which the diffusion coefficient increases with the thermal parallel Kubo number was found. It is determined by the dynamical trapping of the trajectories in the structure of the stochastic potential. The radial diffusion of the electrons with small parallel kinetic energy is influenced by the stochastic electrostatic acceleration along the magnetic field. The process is much more complex due to the parallel trapping in the potential wells which appears besides the perpendicular trapping on the contour lines of the stochastic potential. Due to this parallel trapping, the effect of the stochastic parallel motion is attenuated and the variation of the diffusion coefficient is sensibly weaker than in the case of free parallel motion with the same parallel Kubo number.

Magnetic turbulence

We have studied the transport of collisional particles in stochastic magnetic fields using the decorrelation trajectory method [8], [17]. We have derived analytical expressions for the running diffusion coefficient and for the Lagrangian velocity correlation in terms of a set of deterministic trajectories. They are defined in subensembles of the realizations of the stochastic field as solution of differential (Hamiltonian) equations that depend on the given shape of the Eulerian correlation of the stochastic potential. They are approximations of the subensemble average trajectories and represent the dynamics of the decorrelation of the Lagrangian velocity. Since in general the equations for the decorrelation trajectories cannot be solved analytically, a computer code was developed for this purpose and for determining the running diffusion coefficient for arbitrary values of the four parameters of this problem and for given Eulerian correlation of the potential.

We have shown that this rather complicated triple stochastic process is characterized by two kinds of trajectory trappings and contains two decorrelation mechanisms. The latter are produced by the collisional cross-field diffusion and by the time variation of the stochastic magnetic field.

One of the trapping processes concerns the parallel motion and is determined by collisions which constrain the particles to return in the already visited places with probability one. This parallel trapping leads to a subdiffusive transport in the absence of a decorrelation mechanism. This already known process is recovered by our method. The second kind of trapping concerns the magnetic lines which at large magnetic Kubo numbers wind around the extrema of the vector potential $\mathbf{A}=a\,\mathbf{e}_z$ that determines the stochastic conponent of the magnetic field($\mathbf{e}_z$ is the unit vector along the confining magnetic field). The effects of the magnetic line trapping on the collisional particle transport is studied for the first time. We show that in the absence of a decorrelation mechanism, the magnetic line trapping determines a transient decay of the running diffusion coefficient D(t) appearing before the parallel trapping is effective. The simultaneous action of both trapping processes determine a non-linear build-up of Lagrangian velocity correlation and eventually the parallel motion washes out the effect of the magnetic line trapping. Consequently, the asymptotic behaviour of the running diffusion coefficient is exactly the same as in the quasilinear conditions when the stochastic magnetic field does not generate magnetic line trapping.

The effect of the two decorrelation mechanisms is studied afterwards. We show that the effective diffusion coefficient and its dependence on the parameters result from a competition between the trapping and the decorrelation processes and more precisely from the temporal ordering of the characteristic times of these processes. Each one of the two decorrelation mechanisms leads to the already known diffusion laws when the magnetic line trapping is not present. The trapping of the magnetic lines produces a complicated non-linear interaction between the three stochastic processes which determines new scaling laws of the diffusion coefficient. They appear when the decorrelation time is longer than the flight time but smaller than the correlation build-up time. The first condition ensures the magnetic line trapping and the second prevents the elimination of this trapping effect by the parallel collisional motion. A particularly interesting regime is obtained for collisional decorrelation and consists of an effective diffusion coefficient that decreases when the collisional perpendicular diffusion increases (Figure 1).

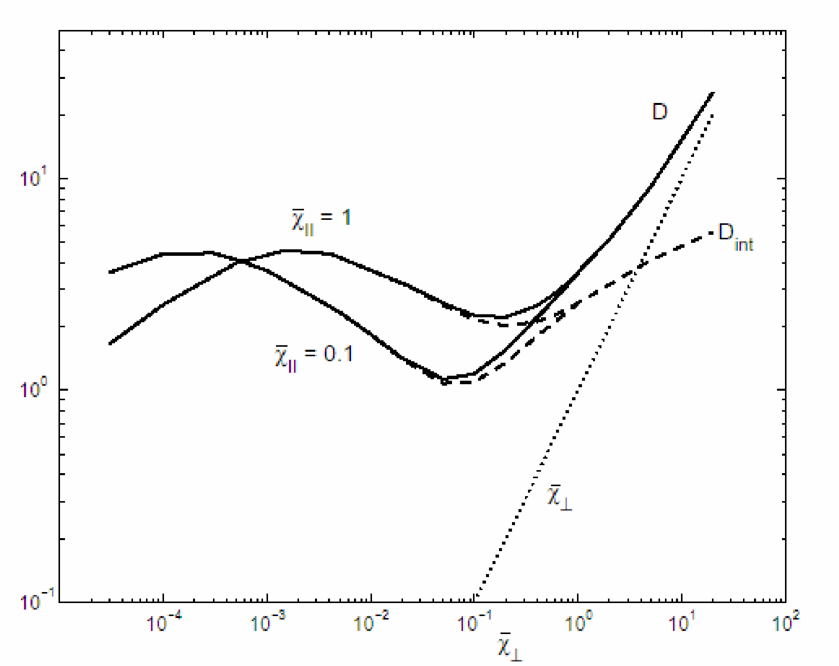

Figure 1. The asymptotic diffusion coefficient as a function of the collisional diffusion coefficient. The total diffusion coefficient D (continuous lines) is compared with the direct collisional contribution (dotted line) and with the interaction term D_{int} (dashed lines) for two values of the collisional parallel diffusivity (see Ref.8).

This rather complex dependence of the diffusion coefficients on the plasma parameters can be used in experiments for controlling the transport. Even without changing the characteristics of the stochastic magnetic field, the diffusion coefficient can be strongly influenced by the parameters which describe particle collisions. A minimum of the diffusion coefficient was obtained for decorrelation times of the order of the average return time for the parallel motion.

2. Model of intermittent transport of energy in the tokamak plasma edge

Instead of using a statistical approach for the characterization of plasma intermittent events, we have developed a detailed study of the topological nature of the individual events (burst of fluctuation). The reason of this fundamental change of perspective consists of the suggestion issued from a Painleve analysis (U. Frisch and R. Morf, Phys.Rev.A23 (1981) 2673) of differential equations that the intermittency is related to the singularities in the plane of complex time. We have proved that these singularities are due to the existence of solutions (instantons) connecting states of distinct topological properties.

Highly ordered states have been obtained by numerical simulations for many important systems in two spatial dimensions: Navier-Stokes, MHD, systems of filaments of vorticity or current, guiding centre particles. These states are described by scalar functions obeying Liouville equation or sinhyperbolic-Poisson equation. The theoretical approach which permitted an explanation of this result is based on the extremum of the entropy over a statistical ensemble of states of a set of discrete objects under the constraints which express the preservation of the invariants of motion. We find that an alternative explanation appears when the fluid model is embedded in a suitable field-theoretical framework. In the new approach the main point is that the asymptotic ordered states have the property of self-duality. In this work we examine several models which have been proposed for fluid-like systems and show that these models can be reformulated such as to exhibit the property of self-duality of the asymptotic relaxational states. In fluid physics the topological quantity is the kinematic helicity $h = \frac{1}{2}\int d^3x \mathbf{v} \bullet \omega$. In more general terms the Chern-Simons action for a potential A (which in general can be an element of a non-Abelian group) is

$$CS(A) = -\int Tr\left(AdA + \frac{2}{3} A \wedge A \wedge A \right)$$

We have shown that the field theoretical framework is able to provide a derivation in analytical terms of the equation obeyed by the scalar stream function ψ in the asymptotical relaxed states of the fluids. This equation is

$$\Delta\psi + \sinh\psi = 0$$

In this way we can understand the origin of this equation and the direction of evolution of a plasma at relaxation.

3. Model for the generation of the transport barriers in rotating plasmas

We have proved that the fluctuation of the ambipolarity condition leads to a fast oscillating rotational motion of the plasma in poloidal direction. This is associated with the presence of a radial flux of electrons that can be caused by a local magnetic fluctuation. We have calculated the frequency and the asymptotic regime, taking into account the poloidal dumping due to the poloidal transit time magnetic pumping.

If the turbulent diffusion of the electrons and ions are different one can expect that a radial electric field develops (so that the ambipolarity is restored) and that poloidal plasma spin-up can occur. However it has been shown that the diffusion of particles in turbulent fields is ambipolar. Itoh (S. Inoue, T. Tange, K. Itoh and T. Tuda, Nucl.Fusion 19 (1979) 12052) consider the difference in the radial fluxes generated by fluctuating fields and the role of the ion polarization drift. Such ”events”, consisting of sudden creation of non-ambipolar fluxes, followed by a fast plasma response in view of restoring the ambipolarity, can appear in a random space and time sequence and on the average can affect the plasma dynamics. To look in detail to only one event we shall adopt an ”initial value” point of view.

We obtain the solution for the time evolution of the poloidal velocity:

$$v(t) = v_0\Theta(t) + v_1 q \frac{d_3}{d_0} + v_1 c \frac{1}{\gamma} \exp(\gamma t)$$
$$+ 2v_1\left(-\frac{d_3\gamma}{d_0} \right) \exp(\alpha_r t)\left[(a_r\alpha_i + a_i\alpha_r)\cos(\alpha_i t) - (a_i\alpha_r - a_r\alpha_i)\sin(\alpha_r t) \right]$$

where $d_0 = \dfrac{\nu}{enB_z}$, $d_1 = \dfrac{1}{\Omega_i} + \dfrac{\varepsilon_0 B_z}{en}$, $d_2 = \dfrac{\varepsilon_0 \nu}{e^2 n^2 \Omega_i}$, $d_3 = \dfrac{\varepsilon_0 m_i}{e^2 n \Omega_i}$, α_r, a_r ,and α_i, a_i are combinations of these physical parameters, given in detail in Ref.[6] .This solution is unexpected because it shows an oscillatory behaviour for the poloidal velocity, a fact which has not been remarked before.

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