THE ANOMALOUS TRANSPORT IN TURBULENT PLASMAS

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1. The evaluation of the anomalous transport fluxes in regions with strong neoclassical effects

1a. The evaluation of the anomalous fluxes in turbulent plasmas with RF heating in a fusion device

In the process of plasma confinement we must take into account the balance between the turbulent transport and the heat power since it may determine the performance of the device. This motivates the evaluation of the anomalous fluxes in turbulent plasmas with RF heating.

A neoclassical theory for the turbulent transport coefficients adapted to a non-maxwellian local distribution function was developed. It specifically considers the case of the Ion Cyclotron Radio-Frequency Heating (ICRH) for two reasons:

First, one may consider in a first approximation, that the electron dynamics is decoupled from the ion dynamics, therefore only the ion fluctuation transport coefficients need to be derived. Second, the quasi-linear heating operator in the Stix representation, as well as the corresponding local equilibrium distribution function, have simple enough expressions so as to allow for analytic computations.

The ensemble-averaged gyro-averaged drift kinetic equation contains the three source terms corresponding to collisionality, rf-heating and electromagnetic turbulence. The reference distribution function is assumed to be slightly different from a Maxwellian,

$$F_0^i(x,\lambda;\xi)=\left(1-\frac{1}{2}\xi+\frac{3}{2}\xi\lambda B^0\right)n_i\left(\frac{m_i}{2\pi T_e(1+\xi)}\right)^{3/2}\exp\left(-\frac{x}{1+\xi}\right)$$

The variables are x the kinetic energy scaled by the thermal energy and λ the ratio of the magnetic moment to the kinetic energy. The parameter Stix ξ describes variation of the distribution function on the heating and B^0 is the local resonance magnetic field.

We evaluate the deviation from the reference state due to turbulence in the presence of RF heating considering that there is no frequency matching between the RF waves and the electrostatic fluctuations. This evaluation is performed for the case of an electrostatic turbulence using the standard model of toroidal confining magnetic field in the limit of zero parallel RF wave-number. The pitch-angle and phase average of this particular expression of the deviation from the non-maxwellian local equilibrium distribution function is expanded in Laguerre-Sonine polynomials. The coefficients of this expansion are related to the parallel fluxes and they contain terms proportional to the electric fluctuations in parallel, perpendicular and poloidal directions.

The reduced transport coefficients for the ions, $C_{s,j}^i$, $j=\|,\perp,n,T,\phi,<P>,B$ (where s =0 correspond to the particle flux and s = 1 to the energy flux), are plotted as functions of the Stix parameter ξ for $\xi\leq 0.4$ for a given aspect ratio $\eta=0.2$ and given safety factor $q=3$. These values of the parameters η and q are in agreement with ITER design parameters. At $\xi \approx 0.3$ the values of the coefficients $C^{i}_{s,j}$ are smaller by a factor 1/10 as compared to their ohmic value (at $\xi = 0$). The coefficients $C^{i}_{s,j}$, $j = \|, \perp$, (which are connected with the electric field fluctuations in the parallel direction, respective perpendicular direction) and the coefficients $C^{i}_{s,j}$, $j = n, T, \phi$, (which multiply the gradients of particle number density, *n*, temperature, *T*, and electric potential, ϕ) are rapidly monotonically decreasing to zero with the Stix parameter. The reduced transport coefficients for the ions $C^{i}_{s,j}$, $j = \langle P \rangle, B$ (which multiply the gradients of RF power density $\langle P \rangle$, and local magnetic field, B) are increasing with ξ up to $\xi \approx 0.1$ and decreasing with ξ for $\xi > 0.1$.

A comparison of the different contributions to the parallel turbulent fluxes is obtained by using the following plasma profiles $\frac{d \ln n}{dr} = -\frac{1}{R_0} \frac{2\eta}{\varepsilon^2 - \eta^2} = \frac{1}{2} \frac{d \ln T}{dr}$ (for the density *n* and temperature *T*) and the magnetic field profile provided by the standard model. Here $\varepsilon = a / R_0$ is the toroidal aspect ratio and, $\eta = r / R_0$ with *r* the minor radius and R_0 the major radius of the magnetic axis. The conclusion is: the turbulent part of the parallel particle and energy flux decrease with an increasing of the Stix parameter (RF power density).

These results [1] were obtained during the periods of secondement at Association EURATOM - Universite Libre de Bruxelles in collaboration with Dr. Boris Weyssow and benefits from fruitful discussions with Prof. Radu Balescu.

1b. About the definition of the Ponderomotive Force

The ponderomotive force (PMF) which appears in the interaction of radio frequency wave (RFW) with plasma can play an important role in producing an internal transport barrier. In order to define the PMF we use the balance equation for momentum. The evaluation of the PMF is made [2] in terms of the characteristics of the RFW using for the rapid oscillating distribution function δf^{α} the formal expression established in the theory of turbulence for a fluctuating distribution function. The relation between the direction of the propagation of RFW and the direction of the PMF action is illustrated for the case of an electrostatic oscillating field $\delta\phi$. In this case the PMF (electrostatic part) $\vec{F}^{\alpha}_{pm,es}$ acting on the α species is given as

$$
\begin{aligned}
\vec{F}^{\alpha}_{pm,es} = & -\frac{n_0^{\alpha} e_{\alpha}^2}{T_{\alpha}} \left\{ \left[\left(\vec{b} \times \vec{\nabla} \ln p_{\alpha} \right) \times \vec{b} \right] + \frac{2\omega}{\Omega_{\alpha}} \left[\vec{k} \times \vec{b} + \left(\vec{b} \times \vec{\nabla} \ln p_{\alpha} \right) \right] \right\} \langle \delta\phi \, \delta\phi \rangle_{ens} \\
& + \frac{n_0^{\alpha} e_{\alpha}^2}{\Omega_{\alpha} T_{\alpha}} \left\{ \left[\left(\vec{b} \times \vec{\nabla} \ln p_{\alpha} \right) \cdot \vec{\nabla} \right] \langle \vec{V}_{\alpha} \rangle - i \left[\left(\vec{k} \times \vec{b} \right) \cdot \vec{\nabla} \right] \langle \vec{V}_{\alpha} \rangle \right\} \langle \delta\phi \, \delta\phi \rangle_{ens} \\
& + \frac{n_0^{\alpha} e_{\alpha}^2}{\Omega_{\alpha} T_{\alpha}} \left\{ \vec{b} \times \vec{\nabla} \ln p_{\alpha} - i \left(\vec{k} \times \vec{b} \right) \right\} \left\langle \delta\phi \left(\langle \vec{V}_{\alpha} \rangle \cdot \vec{\nabla} \right) \delta\phi \right\rangle_{ens} \\
& - \frac{e_{\alpha}}{\Omega_{\alpha} T_{\alpha}} \frac{c}{B_0} \left\{ \vec{b} \times \vec{\nabla} \ln p_{\alpha} - i \left(\vec{k} \times \vec{b} \right) \right\} \left\langle \delta\phi \left[\left(\vec{b} \times \vec{\nabla} \ln p_{\alpha} \right) \cdot \vec{\nabla} \right] \delta\phi \right\rangle
\end{aligned}
$$

Here $\vec{k}$ is the wavenumber vector and ω the frequency of the RFW, $\vec{b}$ is the unit vector in direction of the equilibrium magnetic field, $\vec{V}_\alpha$ is the flow velocity of α species. As we can see, the anisotropy and non-homogeneity of the plasma make that for one given direction of the wave number vector **k,** the PMF $\vec{F}^{\alpha}_{pm,es}$ can have components in all directions (radial, poloidal and toroidal).

Most of these results were obtained during the periods of secondement at Politecnico di Torino, Italy, and benefits from fruitful discussions with Prof. Franco Porcelli.

2. Modeling of anomalous diffusion in Tokamak by iterative applications
Macro task 5.1. Topic: "Validation of physics-based transport models"

General statement of the problem

All of the mathematical models refer to the magnetic field line dynamics in a Tokamak, which are studied by the method of iterative maps. The class of model maps studied was the so-called TOKAMAP [3]

2.1. Definition of the new metric invariants characteristic for anomalous diffusion and elaboration of a C++ program for that

Statement of the problem

In the framework of Ergodic Theory several useful, significant (for anomalous transport in a tokamak, invariants are ***well known****. Nevertheless, these old invariants are incomplete and we studied new, experimentally accessible, invariants.*

2.1.1. New metric invariant and C++ program

We found a useful **new metric invariant, exposed in** [4]. Physically, this invariant is associated to the large time asymptotic behavior of the phase-space volume attained by particles. From the point of view of the theory of the chaotic dynamical systems, this invariant is related to our generalization of the classical theorem of M. Kac, concerning the mean Poincare return time [4]. The advance over the classical theorem of M. Kac is the possibility to compute not only the volume of the ergodic domain, but also the mean values of any observables.

We developed a C++ program to compute this invariant.

On models, described by Standard Map, and TOKAMAP, we found that the return-time asymptotic starts with the main exponential behavior and has a polynomial tail. The domain of validity of exponential behavior increases if the initial domain decreases.

This result show that in a fusion device, like JET. or ITER, in the ergodic domains, the magnetic field line dynamics are a mixture of stochastic behavior with strong mixing, together with a dynamical behavior characterized with very slowly decaying correlations.

In numerical analysis we also used the results developed in [5] and[6].

Conclusion: The proposed objective was attained. The obtained result will be used in the elaboration of the reduced models of the anomalous transport in tokamak plasma for ITER and JET.

2.1.2. Classical Kolmogorov-Sinai dynamical entropy calculations.

Statement of the problem.

We were interested to find a more efficient numerical algorithm to compute the Kolmogorov-Sinai entropy. This quantity is widely used in the characterization of the instability of complex chaotic systems.

Results.

We developed numerical methods for computing the Kolmogorov-Sinai (metric) entropy for dynamical system associated with magnetic field lines in a Tokamak.

The numerical method developed uses new versions of the classical Shannon-MacMillan-Breiman theorem and the Oseledec multiplicative ergodic theorem [4] . We applied this method to the TOKAMAP model, in the work in [7].

This result characterizes short time decay of the correlations in the magnetic field line dynamics in the ergodic domains of ITER and JET.

Conclusion: The proposed objective was attained. The obtained result will be used to speed up the numerical code for tokamak transport simulation, in ITER and JET, in the framework of reduced models.

2.1.3. Systematic exploration of the physical aspect of the mathematical model

Statement of the problem

The phase space structure of the magnetic field line dynamics models is very complex. In particular, r the structure of the domains responsible for transport barriers has a complex fractal structure. Our aim was to investigate this last aspect.

Results

We investigated in a series of works the phase space structure of the mathematical model of the field –line dynamics, the TOKAMAP model. The relation of the mathematical structures and transport barriers was studied in [8,9]

Conclusion: The proposed objective was attained. The apparently mathematical result will clarify about mechanism of the formation of the *transport barriers in JET and ITER.*

2.1.4 Generalization of the definition of the winding number and safety factor for the ergodic domain. Stability and convergence theorems.

Statement of the problem and physical motivation

Up to now, both in the physics and mathematics literature, the winding number and safety factor were not considered as a well-defined quantiiesy in the ergodic domain of the tokamak (or stellarator) plasma. They were defined for nested tori, island, or cantories. The last type of geometrical structures, with very strange fractal structure (terminology linked to Georg Cantor and his fractal, “Cantor dust”, “devil staircase”), was identified in the mathematical literature on area preserving maps as remnants of destroyed magnetic surfaces, or, only viewed as sectioned by a constant toroidal angle plane, as remnants of the invariant circles. When there is no electric field, in the magnetic field line dynamics they act as a semipermeable transport barrier. We elaborated a transport barrier scenario based on cantori in ref [4] and [9]. We intend to eliminate this disparity.

Results [10]**.**

We proved that these quantities could be well defined in a very rigorous mathematical sense. We proved that the counterexamples given in the literature are irrelevant in the framework of the calculations in statistical physics models. In the case of magnetic field configuration close to the integrable case, we proved a *stability theorem*: for small perturbations the variation of the values of the winding number is small.

We wrote a program to compute the winding number in the “Tokamap” model. The program can be used with minor modifications to other model mapping of the field line dynamics. We proved the convergence of the numerical scheme.

Conclusion: The proposed objective was attained. We expect that our result will be used in experimental data processing, at least in the divertor domains of JET and ITER.

2. 2. Realization of a C++ program to characterize the fractal domains

Statement of the problem and physical motivation

In the framework of the magnetic field-line dynamics there are at least two kind of fractal structures: Cantori *and* island around island or planetary systems.

Results

In [11] we investigated the general, global structure of the TOKAMAP model, for physical ranges of parameters. The partial transport barriers or *cantories* was located by using an improved numerical method [12]

We investigated from a statistical point of view the fractal properties of hierarchical islands structures *or filaments [13]. We elaborated a numerical method to study the dependence of the island surfaces on their mean diameter, by using the results obtained previously from the general Kac lemma [4]. We tested this program on the dynamics associated to the Standard Map and completed the study of the TOKAMAP model.*

In a large domain in the interior of the chaotic sea we measured, by numerical experiments, the number of islands, $N(S)$ whose area is less then a given value, S, and $N_1(d)$, the number of islands whose maximal diameter is larger then d. We studied in the limit of small islands the scaling $\log(\log(N(S))) \propto \beta \log(\log(S))$ and $\log(\log(N_1(d))) \propto \gamma \log(\log(d))$. We found $\beta \cong \gamma \cong 1.18 \pm 0.04$. This is consistent with the fact that the chaotic sea is a fat fractal.

Conclusion: The proposed objective was attained. Our result explains qualitatively the slowing down of the radial particle transport, compared with the toroidal and poloidal ones in the tokamak. *The results are related to the filamentary structure of the plasma in fusion devices, like ITER and JET, whose toroidal sections appear as hierarchical, island structures.*

2.3. Iterative Function Systems (Maps) used in modeling the Tokamak and their connection with the diffusive processes

Statement of the problem and physical motivation

In order to obtain statistical information on magnetic field line dynamics we studied TOKAMAP models. The aim was to extract from these models the slow radial dynamics.

Results

Particle loss from confined plasma

We have established the sub-diffusional behavior of the magnetic field lines. We found that the mean value of radial displacement, as a function of number of toroidal turns, grows asymptotically according to a power law, with an exact exponent equal to 1/4, i.e. $\log(R) \propto 0.25.\log(t)$, *where "R" is the mean radius and "t" is the number of toroidal turns.*

These results indicates that the large toroidal turn behavior of the magnetic field lines in the deconfined regime can be described as a simple random walk process, in the radius-squared variable. We explored a simplified stochastic model for particle transport [14], and started the study of a more realistic version, in the final form presented in [15].

Slowing down the internal transport.

We elaborated sound mathematical methods to compute radial particle fluxes through Cantories. Previous results, as well as computer graphics illustration are exposed in [16,17].

Conclusion: The proposed objectives were attained. The obtained result will be used in the elaboration of the reduced models of transport in JET and ITER.

The ref. [18] which contain a review of results on TOKAMAP model was accepted (with results obtained in 2003) and is in press.

Part of the previous results was obtained in collaboration with Prof. R. Balescu and Dr. B. Weyssow, in the period of secondment at U.L.B. (Belgium) in 2003, and Dr. J. H. Misguich, Dr. J.D. Reuss in the period of secondment at C.E.A. Cadarache (France).

3. The diffusion of the magnetic field lines in presence of shear and the effect on the particle diffusion

Topic: "Concept improvements"- Macro task 5.1

3.1. The diffusion of magnetic field lines in the presence of shear (the decorrelation trajectory method) and the influence of the field on the collisional particle diffusion

We have studied the diffusion of the magnetic field lines for a sheared stochastic magnetic field in a slab geometry [19] using the decorrelation trajectory method (DCT) [20]; this method was already used for the electrostatic case, the biased systems and for the collisional particle diffusion in a magnetic field without shear. The model for the stochastic sheared magnetic field have been chosen in slab geometry. The correlations of the magnetic field components are chosen to be Gaussian [21]. At this stage we have analyzed the decorrelation trajectories using the specific parameters describing the system of equations for the sheared magnetic field lines ($\Psi^0 \in [-3,3]$, $b = 1,\ \theta_s = 12.5,\ \alpha = 10,\ \beta = 10^{-3}$) where Ψ^0 is the electrostatic potential value in a specific given subensemble, b is the magnetic field perturbation in the same subensemble; θ_s is the shear parameter, α is the magnetic Kubo number and β is the amplitude of the magnetic field fluctuations) and studied the "frozen" and the "unfrozen" cases. Some of the results (the trajectories and the solution of the system) are represented in Figures 1 and 2. The influence of the shear on the shape of the trajectories is obvious (the last pictures from Figures 1 and 2) and a comparison with the already studied case of the sheared magnetic field in the quasi-linear limit will be done in the framework of the kinetic approach, namely the hybrid kinetic equation [22]. We intend to calculate numerically the non-symmetric running (here the term *running* means the z dependence of the diffusion coefficient; z playing the role of time) and asymptotic diffusion tensor of the magnetic field lines using the Lagrangian velocity correlation tensor obtained by the DCT for different values of the magnetic Kubo number and the shear parameter. We have studied also the decorrelation trajectories for a collisional particle in a stochastic sheared magnetic field [23], by the DCT.

We have represented in Figures 3a and 3b the trajectories, the solution of the system and the hodographs for the following parameters (the same for Figures 3a and 3b): $\Psi^0 \in [-3,3]$, $b = 1,\ \theta_s = 25,\ \beta = 10^{-3},\ \chi_\parallel = 1,\ \chi_\perp = 10^{-2},\ \eta^0 = 1$,(where $\chi_\parallel$ and $\chi_\perp$ are the parallel respectively the perpendicular diffusivities and η_0 is the stochastic parallel velocity in a given subensemble). The others parameters are: $M = 10,\ \alpha = 10,\ L_s = 0.4\,m$ for Figure 3a, and $M = 30,\ \alpha = 30,\ L_s = 1.2\,m$ for Figure 3b. The Lagrangian correlation tensor and the diffusion tensor will be calculated numerically and we expect a saturation of the diffusion for some specific Kubo number, perpendicular and parallel diffusivities. We intend to continue this work in order to re-obtain the results in the quasi-linear limit in the framework of the kinetic approach, namely the hybrid kinetic equation.

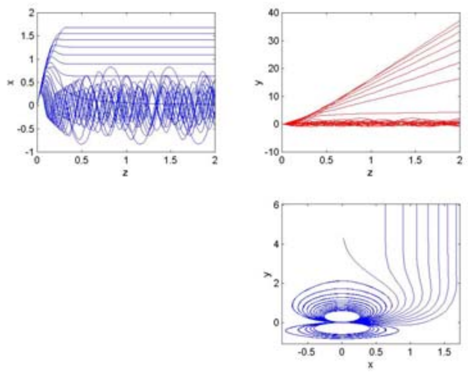

Figure 1 The solutions (first two pictures) and the trajectory for the following parameters: $\Psi^0 \in [-3,3]$, $b = 1$, $\theta_s = 12.5$, $\alpha = 10$, $\beta = 10^{-3}$ in "frozen" case.

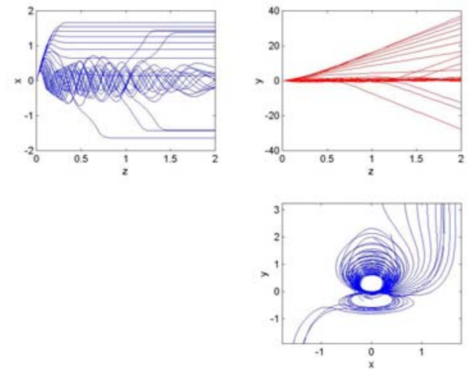

Figure 2 Same as Figure 1 but in "unfrozen" case.

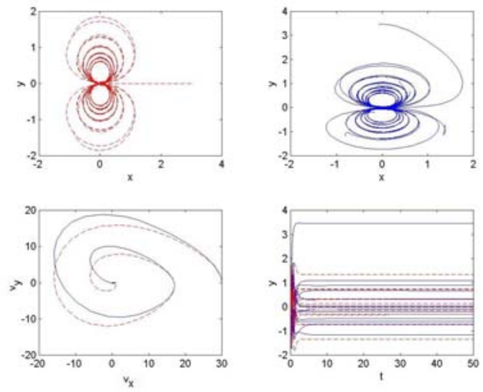

Figure 3a The trajectories, the hodographs solutions of the system for the following specific parameters (in red the case): $\Psi^0 \in [-3,3]$, $b = 1$, $\theta_s = 25$, $\beta = 10^{-3}$, $\chi_\parallel = 1$, $\chi_\perp = 10^{-2}$, $\eta^0 = 1$, $M = 10$, $\alpha = 10$, $L_s = 0.4\,m$

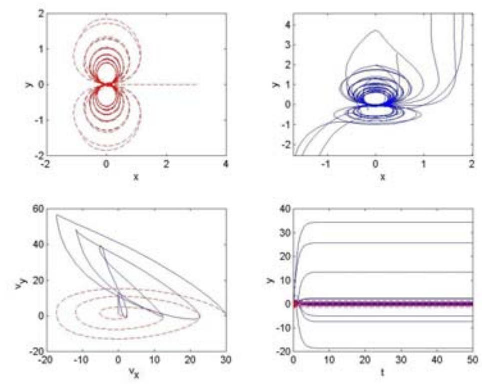

Figure 3b The trajectories, the hodographs and the solutions of the system for the same specific parameters as in Figure 1, shearless except for M and L_s (in red the shearless case): $M = 30$, $\alpha = 30$, $L_s = 1.2\,m$

3.2. The study of the radial electric field in tokamak

We have continued the study of the radial electric field bifurcation due to the following mechanisms of losses: the loss cone loss of ions, the anomalous bipolar loss and the bulk viscosity flux of ions. For some specific parameters entering the flux expressions, a similar behavior for the radial electric field and for the normalized particle flux like in Toda S. et al. in *Plasma Phys. Control. Fusion 38, 1337 (1996)* was observed.

We have used as starting point the paper of Itoh S. and Itoh K. from *Phys. Rev. Lett.* **60**, 2278, 1988 and we have made this analysis studying the dependence of the radial electric field and the normalized particle flux on the control parameter.

Two particular cases for the normalized particle flux are represented in Figures 4 a and 4b for the following specific parameters: $\nu_{*i} = 4$, $\varepsilon = 0.213$, $X_0 = 0.25$, $d = 0.05$ for Figure 4a respectively $\nu_{*i} = 4$, $\varepsilon = 0.213$, $X_0 = 0.25$, $d \geq 0.4$ (where ε is the inverse aspect ratio, X_0 is a reference radial electric field, d is the electron diffusivity and ν_{i*} is the normalized frequency) for Figure 4b. The absence of the bifurcation is obvious in the case presented in Figure 4a while a single bifurcation is present in Figure 4b. We intend to study also the time behavior of the normalized radial electric field considering a specific time variation for the temperature of the ions (a tangent-hyperbolic temporal dependence will be chosen). We intend also to analyze this kind of dynamical equation for a more complete model including other mechanisms of losses.

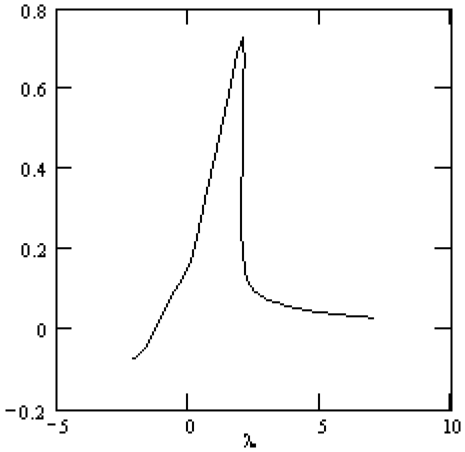

Figure 4a The normalized particle flux following parameters:
$\nu_{*i} = 4,\ \varepsilon = 0.213,\ X_0 = 0.25,\ d = 0.05$

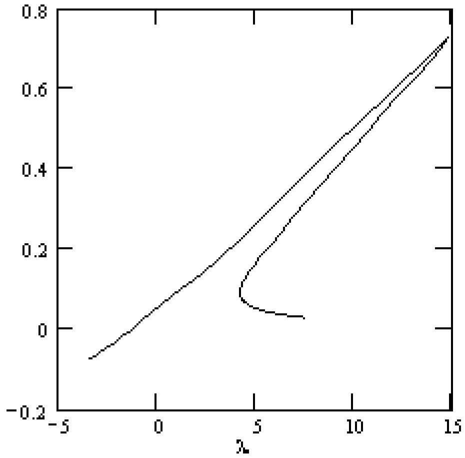

Figure 4b The normalized particle flux for the the following parameters:
$\nu_{*i} = 4,\ \varepsilon = 0.213,\ X_0 = 0.25,\ d \geq 0.4$

These works have relevance to ITER and JET and were done partially during the mobility periods at Cadarache (collaboration with Dr. J. Misguich, Dr. J. D. Reuss), and Universite Libre de Bruxelles (collaboration with Prof. R. Balescu and Dr. B. Weyssow).

4. The description of some diffusion phenomena in TOKAMAK using the fractal geometry

(The description of the chaotic dynamics of some dynamical systems related to the diffusion processes in TOKAMAK)

Topic: "Concept improvements" Macro task 5.1 and 5.2

In order to describe the magnetic field lines' dynamics in tokamak, a discrete dynamical system can be used. Its phase-space is a plane transversal to the torus, at a fixed toroidal angle (in fact a Poincare section) and the succesive intersections of the magnetic field's line with the plane are studied.

In the phase-space the coordinate (θ, ψ) are the poloidal angle $\theta(\bmod 2\pi)$ and the flux coordinate which is related to the radial coordinate r by $\psi = r^2/2$ in dimensionless units.

Our study was focussed on the "Tokamap-model" introduced in [3]. The system is generated by the "Tokamap", namely $T_K : S^1 \times R \to S^1 \times R,\ K \in [0, 2\pi)$ which are given by T_K :

$$\begin{cases} \psi' = \dfrac{1}{2}[\psi - 1 - \dfrac{K}{2\pi}\sin(2\pi\theta) + \sqrt{\left(\psi - 1 - \dfrac{K}{2\pi}\sin(2\pi\theta)\right)^2 + 4\psi}\,] \\ \theta' = \left(\theta + \dfrac{1}{4}(2-\psi')(2 - 2\psi' + \psi'^2) - \dfrac{K}{4\pi^2}\dfrac{1}{(1+\psi')^2}\cos(2\pi\theta)\right)(\bmod 1) \end{cases}$$

In the limit $K = 0$ (the ideal situation) the orbit of any $(\theta_0, \psi_0) \in S^1 \times R$ is contained in the invariant circle $\psi = \psi_0$ and it is periodic or quasiperiodic. By increasing K (which measure the amplitude of the perturbation due to some internal or external factors) the dynamics becomes more and more complicated (the first 500 points of 10 orbits are ploted in Figure 5): some orbits remain regular, but some "chaotic orbits" appear, which wander -apparently in a random manner- in a region bounded by invariant curves. These invariant curves (KAM barriers from a mathematical point of view) act as transport barriers. They correspond to some closed magnetic surfaces in a tokamak. In the chaotic zone some partial transport barriers can be identified. A chaotic orbit can hardly penetrate these partial barriers - which are Cantor-type sets called Cantori - and wander in the hole chaotic zone. These partial barriers appear by a breaking-up process of some barriers.

For both theoretical and practical reasons, the study of the breaking-up process is important. From a practical point of view, the presence of a barrier in the peripheral zone of the poloidal section ensures the confinement of magnetic field lines, which is an important ingredient for the needed plasma confinement.

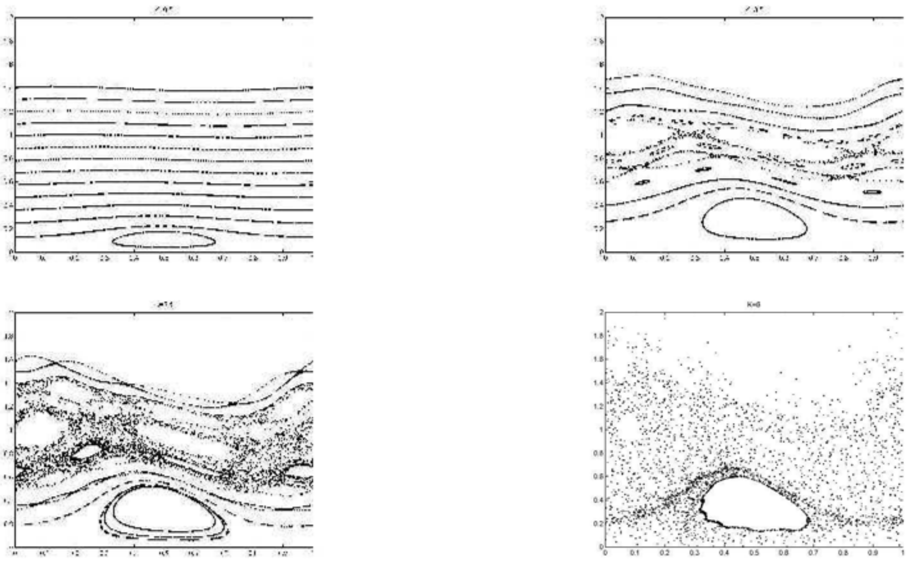

Figure 5 Phase portraits of the tokamak corresponding to various values of the stochasticity parameter: K=0.5, K=3.5, K=4.5, K=6.

For $K = 4.875$, a barrier having the winding number $N(4,2)$ was identified (see [24]) in the peripheral zone of the disk.

We studied the breaking-up process of this barrier. The main result can be founded in [25].

The edge barrier having the winding number $N(4,2)$ is broken for $K = 4.879$. For $K \approx 4.878$ a non-smooth KAM barrier exists in the peripheral zone of the disk. The barrier is smooth for $K < 4,877$.

The threshold for breaking-up of the barrier can be considered $K_c = 4.878$, with an error of the order 10^{-3}.

When $K > K_c$ the barrier is broken and some magnetic field lines escape from the plasma. When $K < K_c$ the external barrier actually confines magnetic field lines inside the plasma.

In the study we used the Greene's criterion and we proved that:

- for $K = 4.877$ both positive and negative residues go to 0, so the subcritical case in Greene's criterion [25] is fulfilled. It results that a smooth KAM barrier exists.
- for $K = 4.878$ the residues sets is bounded, so the critical case in Greene's criterion is fulfilled and a non-smooth barrier exists.
- for $K = 4.879$ the positive residues increase, hopefully to $+\infty$ and the negative residues decrease, hopefully to $-\infty$, so the supercritical case in Greene's criterion is fulfilled

In Figure 6 and Figure 7 the residues R_n^+ (respectively R_n^-) are plotted versus q_n. The different situations can be easily observed.

The main problem in the study is the accurate determination of the periodic points involved in Greene's theory. Because Tokamap is not reversible, the classical techniques cannot be applied. In order to determine the periodic points, we used a very quickly convergent algorithm based on the Fletcher-Reeves method [26]. In spite of its high convergence and of the large number of digits used in calculations (18 digits) the round-off errors still induce numerical deviations and errors in computing long periodic orbits, for the well known reason that hyperbolic points are unstable and the system exhibits a strong dependence on initial conditions.

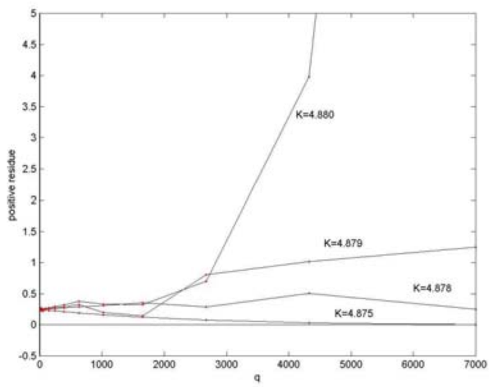

Figure 6 The residues R_n^+ plotted versus q_n

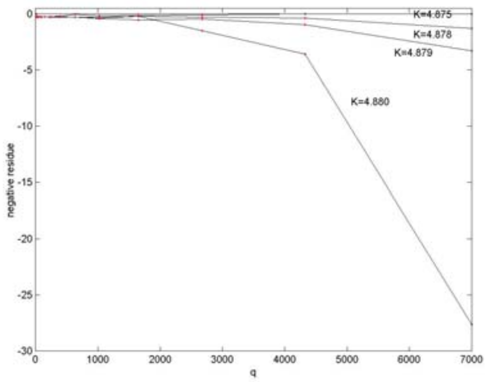

Figure 7 The residues R_n^+ plotted versus q_n

The breaking-up process is not sudden. Using fourteen convergents we can notice that the critical case in Greene's criterion is also fulfilled for $K = 4.8799$ and for $K = 4.8781$. A more precise evaluation of the critical value of K can be obtained by using more terms in the convergents' sequence, but the error cumulation enables us to compute real periodic orbits, whose period is larger than 6008.

The "tokamap model" is a conservative system generated by an area-preserving map. Many fractal objects appear in the phase-space: the chaotic zone (which is a "fat fractal" having the box-counting dimension equal to 2), the internal partial transport barriers (which are Cantor-type sets having the box-counting dimension equal to 0), etc, the boundaries of the stability islands, a self similar island chains structure surrounding the stability islands. Their properties are quite different from the strange attractors' properties in dissipative dynamical systems (for example the usual fractal dimensions are not relevant). The connection between some fractal properties and transport phenomena remains to be studied.

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