THE ANOMALOUS TRANSPORT IN TURBULENT PLASMAS

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1. Effect of RF waves on transport in tokamak plasmas

Topic "Heating and CD"- Macro task 5.1

The evaluation of the anomalous fluxes in turbulent plasmas with RF heating in a tokamak

In the previous work we have evaluated the parallel turbulent flux in a plasma in the presence of ICRH waves (largely used in fusion device for auxiliary heating). As a natural continuation we have evaluated the turbulent radial particle flux in turbulent plasmas in the presence of RF heating[1]. Using a perturbation method we found a formal general solution for the distribution function part dependent on gyrophase angle, which permits the radial particle flux to be expressed. From the different components of the latter we have evaluated the turbulent particle flux in the presence of RF heating.

This formalism was applied to a specific case: evaluation of the turbulent radial particle flux of the ions (one single species) in the presence of the auxiliary heating in ICRH case[2] using a specified local equilibrium distribution function slightly different from a Maxwellian:

$$F_0^i(x,\lambda;\xi) = \left(1 - \frac{1}{2}\xi + \frac{3}{2}\xi\lambda B^0\right) n_i \left(\frac{m_i}{2\pi T_e(1+\xi)}\right)^{3/2} \exp\left(-\frac{x}{1+\xi}\right)$$

The variables are x the kinetic energy scaled by the thermal energy, and λ the ratio of the magnetic moment to the kinetic energy. The parameter Stix ξ describes variation of the distribution function with the heating and B^0 is the local resonance magnetic field.

After a lengthy calculation the following terms of the turbulent radial particle flux of ions have been obtained:

$$\left\langle \Gamma_{0-1}^{i,r} \right\rangle_S = -\frac{1}{B_0} \frac{c e_i n_i}{\pi T_e} \left\langle \delta\phi\, \delta E_{pol} \right\rangle_{ens} C_{0-1}(\xi;\eta,b_0)$$

$$\left\langle \Gamma_{0-2}^{i,r} \right\rangle_S = -\frac{1}{B_0} \frac{e_i n_i}{\pi m_i c} \left\langle \delta A_{\parallel}\, \delta B_{rad} \right\rangle_{ens} C_{0-2}(\xi;\eta,b_0)$$

$$\left\langle \Gamma_{0-3}^{i,r} \right\rangle_S = \frac{1}{B_0} \frac{2 e_i T_e}{m_i^2 c} n_i \left(\frac{m_i}{2\pi T_e}\right)^{3/2} \left\langle \delta A_{\perp}\, \delta E_{pol} \right\rangle_{ens} C_{0-3}(\xi;\eta,b_0)$$

$$\left\langle \Gamma_{1-1}^{i,r} \right\rangle_S = -\frac{1}{B_0^2} \frac{c n_i}{4\pi} \left\langle \delta E_{\parallel}\, \delta B_{pol} \right\rangle_{ens} C_{1-1}(\xi;\eta,b_0)$$

$$\left\langle \Gamma_{1-2}^{i,r} \right\rangle_S = -\frac{1}{B_0^2} \frac{c n_i}{4\pi} \left\langle \delta B_{\parallel}\, \delta E_{pol} \right\rangle_{ens} C_{1-2}(\xi;\eta,b_0)$$

$$\left\langle \Gamma_{2-1}^{i,r} \right\rangle_S = \frac{1}{B_0^2} \frac{c n_i}{4\pi} \frac{1}{R_0} \left\langle \delta E_{\parallel}\, \delta A_{\parallel} \right\rangle_{ens} C_{2-1}(\xi;\eta,b_0,q)$$

$$\left\langle \Gamma_{2-j}^{i,r} \right\rangle_S = -\frac{1}{B_0^2}\frac{cn_i}{4\pi}\left\langle \delta E_{\|}\,\delta A_{\|}\right\rangle_{ens} g_j C_{2-j}\left(\xi;\eta,b_0\right) \ , \ j = 2,3,4,5$$

Where

$$g_2 = \frac{e_i}{T_e}\frac{d\phi}{dr} \ , \ g_3 = \frac{d\ln n_i}{dr} \ , \ g_4 = \frac{d\ln \xi}{dr} \ , \ g_5 = \frac{d\ln B^0}{dr}$$

$$\left\langle \Gamma_{2-6}^{i,r} \right\rangle_S = -\frac{1}{B_0^2}\frac{cn_i}{4\pi}\left\langle \delta B_{\|}\left(\nabla\delta\phi\right)_{pol}\right\rangle_{ens} C_{2-6}\left(\xi;\eta,b_0\right)$$

We have plotted the coefficients C_{s-t} functions of ξ for the values $\eta = \frac{r}{R_0} = 0.1$, $b_0 = \frac{B(\theta_{res})}{B_0} = 0.9$ and safety factor $q = 3$ that agree with the parameters designed for ITER.

In the range $0 < \xi < 1$ we obtain the following results:

(a) $C_{0-1}(\xi), C_{0-2}(\xi), C_{0-3}(\xi)$ have positive values and decrease with increasing of ξ.

(b) $C_{1-1}(\xi), C_{1-2}(\xi), C_{2-1}(\xi), C_{2-2}(\xi), C_{2-6}(\xi)$ have positive values and increase with increasing of ξ

(c) $C_{2-3}(\xi), C_{2-4}(\xi), C_{2-5}(\xi)$ have negative values and their absolute values are increasing with ξ

This evaluation is performed using the standard model of toroidal confining magnetic field. All the quantities have been supposed not to depend on poloidal angle θ except the magnetic field.

This work was done partially during the mobility periods at Universite Libre de Bruxelles (collaboration with Dr. B. Weyssow) and C.E.A. Cadarache where there were benefits of fruitful discussions with Dr. J.Misguich and Dr. Xavier Garbet.

2. Elaboration of numerical and analytical methods for particle dynamics response to perturbations in the plasma core.

Topic: "Validation of physics based transport models, response to perturbations",
Macro-Task 5.1

2a: Stability and convergence results in the framework of Kinetic Theory and Ergodic Theory[3].

This work aims to find some general rules in creating internal transport barriers in a tokamak by lower hybrid current drive. The conclusion is: in the ergodic zone it is possible to create a new transport barrier only if already large islands are present. In previous work[4],[5] concerning the statistical approach on perturbation theory of Hamiltonian dynamical systems, we obtained:

Small perturbations of ergodic dynamical systems give rise to small modifications of the phase space portrait. Generic small perturbations of ergodic dynamical systems, modeling particle dynamics in a tokamak, do not create transport barriers cutting the phase space in two large invariant components. Now, in addition to these results, we obtained:

A1) Numerical experiments clarified the extent of the effect of perturbations on ergodic dynamical systems, modeling particle dynamics in a tokamak. We observed numerically the great resistance under perturbations of ergodic Hamiltonian systems (Kronecker foliations in our numerical experiments) with very weak chaoticity.

This result explains the robustness of the magnetic surfaces with irrational safety factor to a more general perturbation than those expected from classical KOLMOGOROV-ARNOLD-MOSER (KAM) theorem.

A2) Numerical experiments reveal that the large ergodic components of the phase space portrait Hamiltonian maps have stable global properties under small perturbations.

This result, if confirmed by analytic proof, is a statistical analog of the structural stability properties of the chaotic systems with C property.

A3) There exists a statistical version of the KAM theorem, concerning the stability of the phase space portrait of integrable Hamiltonian systems under small perturbations. Note that the stability in a statistical sense is more robust compared to the stability of the individual particle orbits, proved in the classical formulation of the KAM theory.

The results from A1, A2, A3 are relevant for the development of numerical methods for the simulation of the particle transport in the plasma core of ITER and JET, in the analysis of the numerical stability of the algorithms.

B) We obtained a statistical version of the classical perturbation series, for integrable Hamiltonian systems. This statistical perturbation method allows the computation of the matrix elements of the von Neumann projector, between states and observables represented by smooth functions. These matrix elements physically are equivalent to the mixed time and ensemble averages of the statistical observables.

These results are useful in the statistical first principle simulation of the particle and field line dynamics in the plasma core of JET and ITER. It allows replacing the CPU-time consuming simulations, which require numerical solution over a long time of differential equations, by simple analytic approach, or using software like MATHEMATICA or MAPLE.

2b: Numerical simulation of effects of the stochastic perturbations.

This milestone, and the following, is part of an effort to improve the hybrid Particle and Monte-Carlo methods to be used in the integrated numerical codes for the large tokamaks.

We studied numerically the effect of a new class of random perturbations on particle dynamics in a stationary magnetic field. The perturbations originate from models associated to random linear amplification[6], modeling the electrostatic fluctuations. *In the framework of this model we observed a new kind of anomalous transport, both in longitudinal and transverse directions related to the magnetic field lines.* In this type of transport the moments of the velocity of low order are convergent for large time while higher order moments diverge exponentially. It can be confused easily with classical transport if we study only the lower order moments of the particle displacement. We also obtained analytically solvable limiting cases, used for controlling the accuracy of the numerical method. The detailed study of this kind of model is under way.

The investigation was performed in collaboration with Dr. Boris Weyssow from U.L.B. (Belgium) in the period of secondment in 2003.

2c: Speed up of the numerical integration methods.

We elaborated a fast algorithm for the numerical integration of special systems of stochastic differential equation. These stochastic differential equations arise in the self-organized criticality models of the burst processes in tokamak and astrophysical plasmas. In general, the numerical simulation of transport processes described by stochastic differential equations, it is difficult to obtain general high-order numerical schemes. We overcame this difficulty by restricting our study to a limited, but physically interesting case: linear stochastic differential equations and guiding center equations with random electric field.

We used our new result, on the super diffusive fractional Brownian motion, which in our models plays the role of the long time memory processes, triggering the burst particle avalanches in the turbulent tokamak plasma.

We also elaborated a numerical algorithm for the fast computation of dynamical invariants (dynamical entropy and phase space volume of ergodic components) of deterministic chaotic systems[7].

The results were obtained in collaboration with J. H. Misguich and J. D. Reuss from C.E.A. Cadarache, France, in the period of secondment in 2003.

2d: Analytic treatment of the effects of quasi-periodic perturbations.

The aim of this study is to revise the *self organized criticality* (SOC) *[e.g. I.. Gruzinov, P. H. Diamond, M. N. Rosenbluth in Phys. Rev. Letters, 89, (2002), 255001]* concepts in tokamak turbulence. The SOC concepts in turbulence studies were subjected in recent years to an **intense criticism**, because SOC models predict either: a) Gaussian fluctuations with algebraic correlation decay, or b) Large non-Gaussian fluctuations with exponential correlation decay.

But experiments, simulations and astrophysical data show non-Gaussian fluctuations with algebraic correlation decay.

Our model assumes that the small fluctuations are of type a) and difficult to observe. But these small amplitude fluctuations act as a multiplicative noise, which produce large, non-Gaussian fluctuations, with heavy tail, which appear as a dominating effect in experiments. The correlation decay is also algebraic.

Our result can be viewed as a strong support that SOC concepts can still be used in the study of tokamak plasma turbulence.

We studied analytically, by statistical methods, the effects of the random quasi-periodic perturbations, with a random frequency spectrum, on the linearized versions of dynamical systems. We considered the special case of linear dynamical systems, which in the unperturbed state are stable but close to the stability threshold. We supposed that the perturbations are represented by a linear combination of colored noises, which acts on the system as *multiplicative noise, as well as additive noise.* Our model is very general and contains the limit cases when the multiplicative and additive noises are strongly (anti) correlated as well as the case when the noises are completely decorrelated.

We obtained an explicit, simple formula for the heavy tail exponent of the probability distribution function of the edge plasma fluxes.

By using the self-similarity hypothesis, this model explains the extremely low value of the heavy tail exponent of the probability distribution of the outgoing particle fluxes, found in experiments on the DIII- D tokamak.

The model explains also the algebraic, very slow, decay of the correlations, found experimentally on the same DIII D tokamak, in astrophysical data on solar activity, and numerical simulations in the framework of the ***shell model****.*

Our model give a quantitative result that is very robust, i.e. the very low value of the heavy tail exponent is insensitive to the numerical values of the parameters of the model.

The self-similarity hypothesis used in our model clarifies the mathematical structure of the random multiplicative noise: its long time characteristics are similar to a fractional Brownian motion, with a Hurst exponent H>0.5. We obtained also an unexpected mathematical result: the persistent (or superdiffusive) fractional Brownian motion can be generated by a linear superposition of an infinite set of independent coloured noises, whose correlation time spectrum is self-similar. This result is very useful for future numerical simulations of the stochastic models of transport in a turbulent plasma: it allows thedevelopment of a fast generator for fractional Brownian motion.

Conclusion: Our result opens the possibility to study complex tokamak plasma turbulence by a new method, which preserves many features of the SOC models. *Can be adapted to the study of the Edge Localized Modes in JET or TORE-SUPRA.*

The results were obtained in collaboration with Dr. Boris Weyssow from U. L. B. (Belgium), in the period of the secondments in 2003.

3. The diffusion of the magnetic field lines in presence of the shear and the effect on the particle diffusion

Topic: "Concept improvements"- Macro task 5.1

3.1. The diffusion of the magnetic field lines in presence of the shear

We have finished the study of the diffusion of the magnetic field lines for a sheared stochastic magnetic field in a slab geometry[8] using the decorrelation trajectory method (DCT)[9]; this method was already used for the electrostatic case, the biased systems and for the collisional particle diffusion in a magnetic field without shear. The model for the stochastic sheared magnetic field has been chosen in slab geometry. The correlations of the magnetic field components are chosen to be Gaussian[10]. We have calculated numerically the components of the running diffusion tensor (the z dependent diffusion coefficient; here z plays the role of "time") using the Lagrangian velocity correlation tensor obtained by the DCT and we have analyzed the diffusion of the magnetic field lines for different values of the magnetic Kubo number $\alpha \in [0.05, 10]$ and the shear parameter $\theta_{shear} \in [0.2, 10]$. We have used the specific parameters describing the system of equations for the sheared magnetic field lines (e.g., $\Psi^0 \in [-3,3]$, $\beta = 10^{-4}$) and studied the behavior of the asymptotic diffusion coefficient components. In Figure 1, the diagonal asymptotic diffusion coefficients as functions of the magnetic Kubo number are represented.

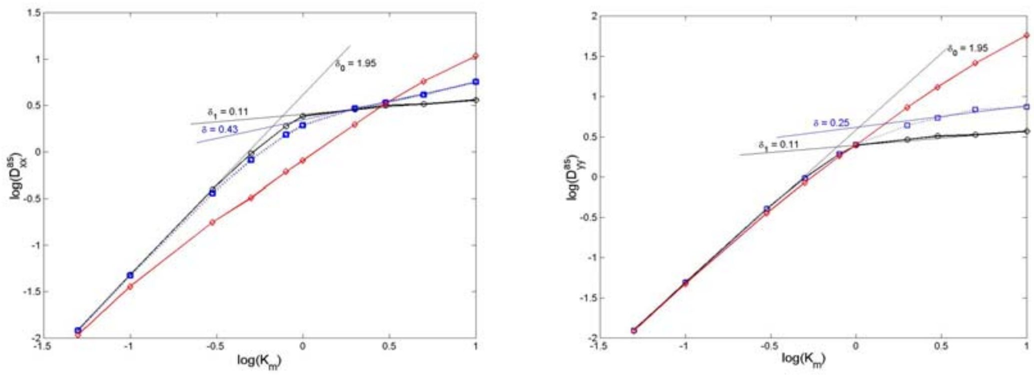

Figure 1 –The diagonal asymptotic diffusion coefficient D_{xx} (left picture) and D_{yy} (right picture) as functions of Kubo number in a log-log plot for different values of shear parameter (θ_s=0 – black line; θ_s=1 – blue line and θ_s=6 – red line) A comparison is made with the results obtained in the quasi-linear approximation and the purely electrostatic case.

3.2. The study of the radial electric field in tokamak

We have continued the study of the radial electric field bifurcation due to the following mechanisms of losses: the loss cone loss of ions, the anomalous bipolar loss and the bulk viscosity flux of ions in collisional and collisionless limits. For some specific parameters entering the flux expressions a similar behavior for the radial electric field and for the normalized particle flux as in Toda S. et al. in *Plasma Phys. Control. Fusion* 38, (1996), *1337*, was observed. We have used as starting point the paper of Itoh S. and Itoh K. *[Phys. Rev. Lett.* 60, (1988), *2278]* and we have made this analysis studying the dependence of the radial electric field and the normalized particle flux on the control parameter.

We have studied also the time behavior of the normalized radial electric field considering a specific time variation for the temperature of ions (a tangent-hyperbolic temporal dependence was chosen)[11]. A decreasing regime followed by a saturated one for the radial electric field time behavior was obtained. This kind of saturation indicates that for such a model, the bifurcation of the radial electric field exists, and the stationary case is recovered. In Figure 2 the radial electric field time-behavior in the collisional (for two different values of the ion effective collisionality) and collisionless cases are represented.

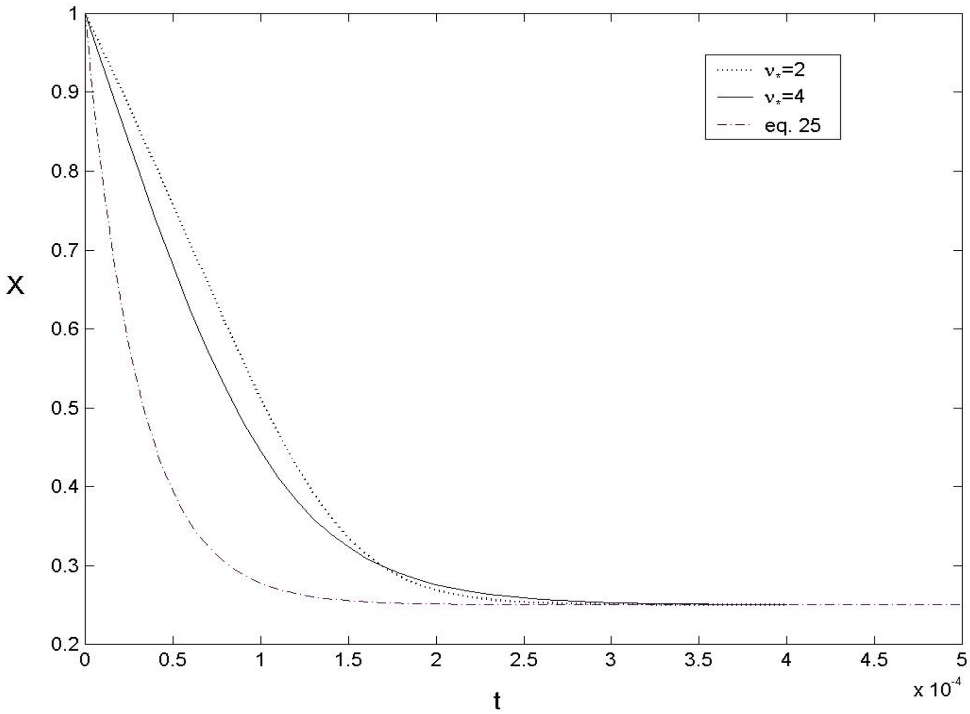

Figure 2. The radial electric field as a function of time in the collisional case for ν_*=2 (dotted line) and ν_*=4 (solid line) and collisionless case (dash-dot line).

3.3 The study of the influence of trapping on the diffusion of a quasi-particle (drift wave packets) in turbulent plasma using the decorrelation trajectory method

We have elaborated a general numerical code in order to solve the above-mentioned problem and we have taken into consideration different solvers (modified Euler, standard Runge-Kutta, Runge-Kutta-Fehlberg (RKF45), predictor-corrector methods) in order to compare the results, the total errors and the time necessary to achieve our final task: the analysis of the running and asymptotic diffusion coefficients; these coefficients depend on many parameters and their calculus need the numerical estimation of 5– fold integrals (for Lagrangian correlation tensor)

or 6– fold integrals (for the running diffusion tensor). This general numerical code based, after our analysis, on the RKF45 method is able to solve practically any specific problem related to the anomalous transport studied by the decorrelation trajectory method.
Until now we have calculated the dependence of the asymptotic diffusion coefficient as a function only of the electrostatic Kubo number K in the unfrozen case corresponding to the basic system of equations given in R. Balescu, *Phys.Rev.E 68, 046409, (2003)*. For example, for K_d = 1 (where K_d is the diamagnetic Kubo number), for the diffusion in (x-x) space, we have obtained for $K < 1$ the dependence scaling as $D_{as} \approx K^{\gamma}, \gamma = 2$ and for $K \leq 7$, $D_{as} \approx K^{\gamma}, \gamma = 0.22$. Until now we can compare only the exponents corresponding to the pure electrostatic case and our model: in the quasilinear regime ($K < 1$) they are both equal to 2. For strong turbulence ($1 < K < 7$) in our model $\gamma = 0.22$ and in the pure electrostatic case $\gamma = 0.67$. For K_d = 0.5, the quasilinear regime is unmodified and the strong turbulence regime is slightly modified. The study must be completed because of the dependence on the diamagnetic Kubo number also; until now we have made all the calculations considering the diamagnetic Kubo number (K_d) only equal to 1 or 0.5; a global study (including $K_d \neq \{0.5, 1\}$) requires a detailed scanning of the parameter space [see R. Balescu, *Phys.Rev.E 68, 046409, (2003)*].
In our case for the running diffusion tensor, (the time-dependent diffusion coefficient), (and also for the Lagrangian correlation tensor) we obtained 16 different components in comparison with one component in the pure electrostatic case and 2 components in the biased case.
This detailed analysis requires a parallel computer using our code and even in this case the time consumed is very significant.
For our analysis at least 1200 hours/processor are necessary.
This study is still in progress and a full comparison with pure electrostatic and biased turbulence will be done after a global study of the problem.
These works have relevance for ITER and JET and were done partially during the mobility periods at Cadarache (collaboration with Dr. J. Misguich, Dr. J. D. Reuss) and Universite Libre de Bruxelles (collaboration with Prof. R. Balescu and Dr. B. Weyssow).

4. Hamiltonian mappings (Mathematical aspects of Internal Transport Barriers in the presence of reversed shear -non twist map- and magnetic reconnection phenomena)
Topic: "Concept improvements" Macro task 5.1 and 3.2
In the previous work the transport barriers were identified and described in two Hamiltonian models for the magnetic field lines dynamics in TOKAMAK (namely the Tokamap and the Rev-Tokamap models). The reconnection of the unstable manifolds of some hyperbolic periodic points was studied in reversed shear situations (the Rev-Tokamap model, corresponding to a non-monotonic q-profile). This phenomenon corresponds to magnetic reconnection. The paper[12] (submitted for publication in 2002 and published in 2003) contains the main results.
The milestones for the reported activity were:
1). To specify an analytical method for locating the (previously observed) transport barrier surrounding the shearless curve in reversed shear models (generated by non-twist area-preserving maps) for the magnetic field lines' dynamics in TOKAMAK
2). To study the breaking-up process of the (previously located) transport barriers in the reversed shear model (the Rev-Tokamap model) for various q-profiles and various values of the stochasticity parameter.
3). To study -using the mapping technique- the passing particles' motion in the same magnetic configurations (the Tokamap and the Rev-Tokamap models).

4). To apply some recently developed mathematical techniques in order to control (reduce) the radial diffusion in the Tokamap model.
We present now the main obtained results:

1**) An analytical method for locating the non-twist annulus surrounding the shearless curve (corresponding to a zero magnetic shear surface in TOKAMAK) in general non-twist dynamical systems**

The non-twist area-preserving maps' prototype is the application $T_k : [0,\infty)\times S^1 \to [0,\infty)\times S^1, T_k(\psi,\theta) = (\overline{\psi},\overline{\theta})$ obtained from the system $\begin{cases} \overline{\theta} = (\theta + W(\overline{\psi}) + kg(\theta)h'(\overline{\psi})) \pmod 1 \\ \overline{\psi} = \psi - kg'(\theta)h(\overline{\psi}) \end{cases}$. Here $g : S^1 \to R$ and $h:[0,\infty) \to R$ are given functions.

The winding function $W:[0,\infty) \to R$ is a convex function having a minimum.

The shearless curve is the closure of an orbit having the minimum rotation number.

In computer simulations a regular zone surrounding the shearless curve was observed. This zone separates two chaotic zones in the phase space and cannot be traversed by any orbit. It acts as a transport barrier. As far as we know an analytical method for the location of this transport barrier was not yet formulated. We proved the following results:

The points of the non-twist curve $C_n : \frac{\partial\overline{\theta}}{\partial\theta}(\psi,\theta)\cdot\frac{\partial\overline{\theta}}{\partial\psi}\left(T_k^{-1}(\psi,\theta)\right) + \frac{\partial\overline{\theta}}{\partial\psi}(\psi,\theta)\cdot\frac{\partial\overline{\psi}}{\partial\theta}\left(T_k^{-1}(\psi,\theta)\right) = 0$ have regular orbits. The closure of their orbits forms the non-twist annulus surrounding the shearless curve.

- The images of the (geometrical) circles $\psi = \psi_0$ that intersect the critical vertical curve $\Gamma_v : \frac{\partial\overline{\theta}}{\partial\theta}(\psi,\theta) = 0$ are not invariant circles but have the folding property. We denote by $(\psi^*, 1/2)$ the intersection of Γ_v with the line $\theta = 1/2$ and $N = \left\{(\psi,\theta) \mid \psi^* < \psi, \frac{\partial\overline{\theta}}{\partial\theta}(\psi,\theta) < 0\right\}$. The region N cannot be crossed by a rotational invariant circle if $T_k\left(\psi,\frac{1}{2}\right) = (\psi,\overline{\theta})$. The orbits of their points are periodic (all are elliptic points) or chaotic. The value ψ^* is a bound of the chaotic zone on the line $\theta = 1/2$.

These results were applied for locating the transport barrier in the Rev-Tokamap model. This model was proposed in 1998 by R. Balescu in order to describe the dynamics of magnetic field lines in TOKAMAK in a reversed shear situation: the winding number (the inverse of the q-profile) has a maximum value. This model can be obtained from the prototype for

$$W(\psi) = w(1 - a(c\psi - 1)^2),\quad g(\theta) = -\frac{1}{4\pi^2\cos(2\pi\theta)},\ h(\psi) = \frac{\psi}{\psi + 1}.$$

Various q-profiles and various values of the stochasticity parameter were considered.

In each case the stochastic zone and the non-twist annulus were located and the results were compared with the ones obtained by direct computer simulation. The structure of the non-twist annulus was also analysed. The main results were submitted for publication [13].

2). **The study of the transport barrier's breaking-up process**

In order to study the break-up process of the transport barrier surrounding the shearless curve we considered two scenarios:

a). To increase the stochasticity parameter for a fixed q-profile (proposed in the original model by R. Balescu).

In this situation ψ^* is a decreasing function of k and the diameter of the bounded component of Γ_v increases as k increases so the inner and the outer edges of the barrier approach each other and thus the destruction of the rotational invariant circles occurs from those with lower values of the rotation number towards the shearless circle.

There are two thresholds k_T, k_l of the (magnetic) perturbation k, such that for $k_T < k < k_l$ there is a barrier in the domain of physical relevance $0 \le \psi \le 1$, sharply separating a quasi-regular region from a completely chaotic one. For $k < k_T$ the regular zone actually extends down to the magnetic axis and no inner edge of the barrier can be easily identified. For $k > k_l$ no invariant circle exists, all the chaotic orbits exhibit an unbounded radial motion and their confinement is compromised. For example, in the case $w = 0.67, w_0 = 0.3333, w_1 = 0.1667$ we obtained $k_T = 2.735$ and $k_l = 6.20$.

b). To fix the stochasticity parameter and to modify the q-profile (by decreasing its minimum value).

We considered fixed values for $k,\ w_0, w_1$ and we increased the maximum value w of the winding function. We observed that the width of the transport barrier seems to have a maximum near the main rationals. This effect was observed in experiments and must be explained from a theoretical point of view.

The main results will be submitted for publication[14].

3). **Control in the Tokamap model**

In order to limit the radial motion of some magnetic field lines and to confine them in a bounded region of phase space, some mathematical control techniques were applied in a generalized model (including both the Tokamap and the Rev-Tokamap case). It was proved that some usual techniques (Kwon, Phys. Lett. A, 1999 or Y. Zhang, Phys. Rev. E 2000) stabilize some chaotic orbits but destroy all the transport barriers of the initial system. The controlled system is dissipative and it is not a reasonable model for the magnetic field lines in TOKAMAK. A localized control (acting only on the control disk - small disk whose centre and radius are imposed) was proposed in[15].

The Tokamap model is obtained from the previously presented prototype for

$$W(\psi) = w(1 - a(c\psi - 1)^2),\quad g(\theta) = -\frac{1}{4\pi^2 \cos(2\pi\theta)},\ h(\psi) = \frac{\psi}{\psi + 1}.$$

The control disk is $D_c = \{(\psi,\theta) \mid d((\psi,\theta),(\psi_0,\theta_0)) < r\}$ and the control term has Gaussian form in one dimension. It is

$$f(\psi,\theta) = p \cdot \begin{cases} e^{-\frac{d^2((\psi,\theta),(\psi_0,\theta_0))}{r^2 - d^2((\psi,\theta),(\psi_0,\theta_0))}} & if\ d((\psi,\theta),(\psi_0,\theta_0)) < r \\ 0 & if\ d((\psi,\theta),(\psi_0,\theta_0)) \ge r \end{cases}$$

The control parameter is p. For various positions of (ψ_0, θ_0) and r we stabilised a large part of the chaotic orbits starting from $[0,3] \times S^1$.

(ψ_0, θ_0)	r	p	Chaotic orbits	Stabilized orbits

-	-	-	1224	-
(2.0, 0.1)	0.1	-0.1	827	33,52%
(1.5, 0.5)	0.1	-0.1	622	51.00%
(2.0, 0.9)	0.1	-0.1	514	58.60%

The controlled system is compatible with the toroidal geometry, stabilizes a large part of the unbounded chaotic orbits and preserves the transport barriers that do not intersect the control disk. Even if it is a theoretical approach, the application of this control effectively improves the magnetic field lines confinement, a very important factor in plasma confinement.
The main results are contained in[16] and will be submitted for publication.

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