INTERPRETATION AND CONTROL OF HELICAL PERTURBATION IN TOKAMAKS

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During the period January-December 2007, the theoretical and modelling research activity of the "**Mathematical Modelling for Fusion Plasmas Group**" of the National Institute for Laser, Plasma and Radiation Physics (NILPRP), Magurele - Bucharest, Romania was focalized on:

- ***Plasma models for feedback control of helical perturbations (Resistive wall modes)*** This activity was performed in collaboration with the Max-Planck - Institut für Plasmaphysik (IPP), Tokamakphysics Department, Garching, Germany, and represents a continuation of our activity from 2006.

1. Resistive wall modes

It is known that the maximum achievable β in "advanced tokamaks" is limited by the pressure gradient driven ideal external-kink modes (10^{-6}s). When a tokamak plasma is surrounded by a close fitting resistive wall, the relatively fast growing ideal external kink modes (EKM) is converted into the far more slowly growing "resistive wall mode" (RWM) which grows on the characteristic $\tau_w = L/R$ time of the wall and has virtually identical stability boundaries to those of the EKM in the complete absence of a wall. Note that the stabilization of RWM in ITER, where it is probably not possible to maintain a very fast plasma rotation is still an open problem.

The objective of our common research was the Development of plasma models for feedback control of helical perturbations. To accomplish this objective, we have considered as necessary to develop a **2D numerical model** for the RWM investigation, consisting of a real axisymmetrical tokamak model with arbitrary cross-section and plasma parameters, and with 3D poloidal and toroidal disposals of wall (in its thin wall approximation).

1.1 The 2D numerical model

For this case, we have considered a 2D axisymmetrical equilibrium with a 3D perturbation given by an *m/n=3/2* external kink mode.

The following ***milestones*** have been achieved:

- description of the vacuum field given the normal component of the perturbed field on the plasma boundary for a 2D axisymmetrical tokamak;
- calculation of the surface current corresponding to a given plasma mode;
- calculation of the patterns of the induced eddy currents in the resistive wall.

In contrast to the usual ideal MHD treatment with the extended energy principle, the present analysis is generally non-selfadjoint and includes the necessary perturbed magnetic pressure for satisfying pressure balance. One solving method could be to found a new basis of orthogonal eigenvectors to ensure the self-adjointness property of the energy operator - the normal mode approach [1-2], or to replace a perfect conducting wall with one of finite conductivity and to deduce a modified energy principle [3]. Numerical MHD stability studies in the presence of toroidal rotation, viscosity, resistive walls and current holes, by using the CASTOR FLOW code are presented in Ref. [4]. Modelling of RWMs has been made by using the VALEN code [5] with an equivalent surface current model for the plasma. Extensive study and theoretical development has also been presented by Bondeson and his co-workers [6-7] by using the MARS code. In a first stage, we have adopted the second approach [3], based on the assumption that the plasma displacement trial function, in the situation where a conducting wall located at finite distance from the plasma is considered to be identical to the one used in the evaluation of the potential energy when the conducting wall is moved infinitely far away. We considered such assumption too limiting and have developed another approach.

Writing the expression for the potential energy in terms of the perturbation of the flux function, and performing an Euler minimization, we have obtained a system of ordinary differential equations in that perturbation [8]. This system of equations describes a tearing mode or an external kink mode, the latter if the resonance surface is situated at the plasma boundary. Usually, vanishing boundary conditions for the perturbed flux function at the magnetic axis and at infinity are considered. From single layer potential theory, we have developed an approach to fix "natural" boundary conditions for the perturbed flux function just at the plasma boundary, replacing thus the vanishing boundary conditions at infinity [8]. Now, in the presence of a resistive wall, the boundary conditions of the external kink mode at the plasma boundary are determined by the reciprocal interaction between the external kink perturbation of the plasma and the toroidal wall (in its thin wall approximation). A general toroidal geometry has been considered. By using the concept of a surface current [9], the description and calculation of the influence of the modes outside the plasma were greatly simplified. The normal component of the magnetic field perturbation, at the plasma boundary, has been considered as excited by toroidally coupled external kink modes and it is that component that gives the normal to the wall component of the exciting field causing the wall response via the induced eddy currents.

Our results are devoted to diverted tokamak configurations and will be applied to the ASDEX Upgrade, JET and ITER tokamaks.

General expression of the potential energy

By writing, in a coordinate system with straight field lines (a, θ, ζ) the general expression of the potential energy W in terms of two test functions $u(a, \theta, \zeta)$ and $\lambda(a, \theta, \zeta)$

$$\psi = \sum_{m=-\infty}^{\infty} Y_m(a) e^{i(m\theta - N\zeta)},\ u = \sum_{m=-\infty}^{\infty} U_m(a) e^{i(m\theta - N\zeta)},\ \lambda = \sum_{m=-\infty}^{\infty} \Lambda_m(a) e^{i(m\theta - N\zeta)},$$

instead of the displacement ξ, one obtains the expression of the potential energy [8]:

$$W = \frac{1}{2\mu_0} \int da d\theta d\zeta f(g_{ik}^r, \psi, \Psi, \Phi, u, \lambda, p, F, J) \tag{1}$$

where ψ is the perturbation of the flux function Ψ, g_{ik}^r are the metric coefficients, Φ is the toroidal flux, u and λ correspond to the perpendicular and parallel displacements, respectively, while p, F, J are the pressure, poloidal and toroidal current densities, respectively. After developing the perturbed values in Fourier series, performing an Euler minimisation of the energy functional and then integrating with respect to the angles θ and ζ the result of that minimisation, one obtains a system of coupled ordinary differential equation of the form

$$\mathbf{Y}'' = \mathbf{f}^{-1} \cdot (\mathbf{G} \cdot \mathbf{Y} + \mathbf{V} \cdot \mathbf{Y}') \qquad (2)$$

where **f** **,** **V** and **G** are matrices, and **Y** is the flux function perturbation vector, with the non-diagonal terms representing both toroidicity and shape coupling effects. Close to the magnetic axis *(a→0)*, we have found the following behavior of the amplitude of the flux function perturbation

$$Y^m(a) \sim a^m - \frac{2a^{m+2}}{(m+1)a_{m/N}^2} - \frac{(m-1)a^{m+4}}{(m+1)(m+2)a_{m/N}^4}, \qquad (3)$$

where $a_{m/n}$ is the resonance radius corresponding to the wave numbers *m* and *n*. We have to consider a "natural" boundary condition just at the plasma boundary [8]. To our knowledge, we were the first to develop the methodology to consider, in a flux coordinates system, the boundary conditions just at the separatrix,. From potential theory we know that a continuous surface distribution of simple sources extending over a not necessarily closed Liapunov surface ∂D and of density $\sigma(q)$, generates a simple-layer potential at *p*, in ∂D. After some tedious calculations, the boundary condition becomes

$$\mathbf{Y}_{k+1} = \left(\mathbf{I} - h\mathbf{F} \cdot \mathbf{D}^{-1} - \vec{\alpha}_k\right)^{-1} \cdot \vec{\beta}_k, \qquad (4)$$

with **I** the unit matrix and *h* the "radial" integration mesh, **D** and **F** [*M x M*] complex matrices, with the elements given by the metric coefficients, normal and tangential magnetic field components. α_k is a known [*M x M*] coefficient matrix and β_k a known [*M*] coefficient vector, both resulting from a forth-order Runge-Kutta integration scheme. For unit perturbations $Y_{2/1}$ $(m = 2,\ n = 1)$ and $Y_{3/2}$ $(m = 3,\ n = 2)$, the corresponding surface charge distributions are given in Fig.1.

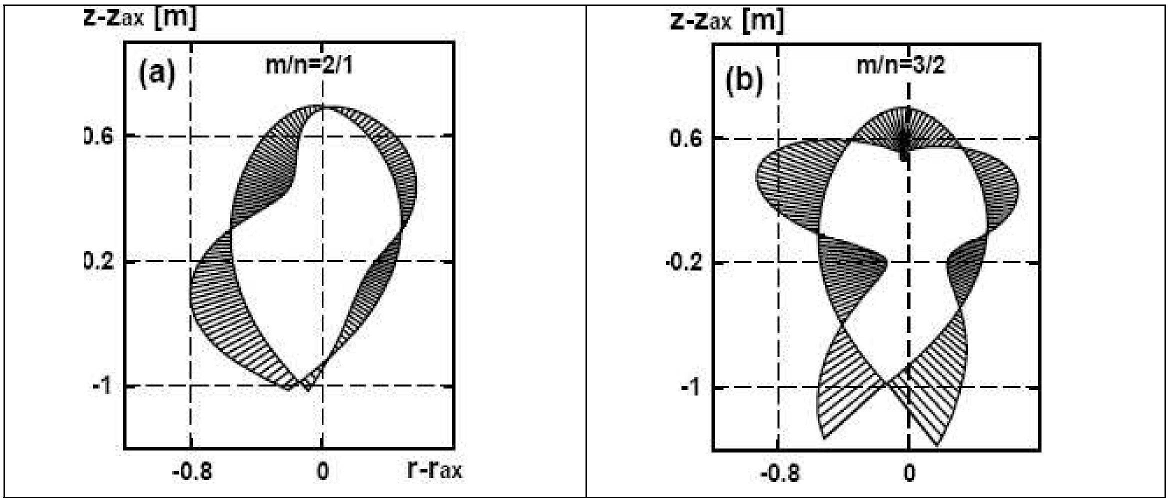

Figure 1. The surface charge distribution along the plasma boundary for unit flux perturbations Y2/1 and Y3/2, respectively. The plasma configuration of the ASDEX Upgrade tokamak corresponding to the shot no. 13476 at 5.2 s has been considered.

Determination of the vacuum field due to a helical perturbation

The perturbation field **B**=*grad Φ* outside the plasma can be assumed to be produced by the surface current equations

$$[\tilde{\mathbf{B}} \cdot \nabla a] \quad \nabla a \times [\tilde{\mathbf{B}}] = \mu_0 \mathbf{K} = -\mu_0 \nabla s \times (\nabla \kappa + \nabla \times \mathbf{g}) = -\mu_0 \nabla a \times \kappa, \qquad (5)$$

where means the jump across the plasma surface. $\kappa(\theta, \zeta, t)$ being the time-dependent stream function of the surface current, and **g** an arbitrary vector. In terms of the external and internal magnetic scalar potentials (with respect to the plasma boundary) one has

$$\mu_0\kappa = -\Phi_e + \Phi_i = -\int_{\partial D} g(\mathbf{p},\mathbf{q})\sigma_e(\mathbf{q})dq + \int_{\partial D} g(\mathbf{p},\mathbf{q})\sigma_i(\mathbf{q})dq \qquad (6)$$

The normal component of the perturbed magnetic field has been considered as excited by a flux function perturbation ψ of unit amplitude $Y(1)$ on the plasma boundary and resulting from an external kink mode. B_n can be calculated with the relation

$$\tilde{B}_n = \frac{\nabla a}{|\nabla a|}\cdot\tilde{\mathbf{B}} = \frac{1}{\sqrt{g_{22}g_{33}}}\frac{\partial\psi}{\partial\theta}, \quad \psi = \sum_m Y_m(a)\exp[i(m\theta - n\zeta)]. \qquad (7)$$

Determination of the diffusion equation in the wall

In an orthogonal curvilinear coordinate system (u, v), with h_u and h_v the Lamé coefficients, the diffusion equation for the eddy current stream function $U(u, v, t)$ in a thin wall looks like [10, 11]

$$\frac{1}{h_u h_v}\left[\frac{\partial}{\partial u}\left(\frac{h_v\partial U}{h_u\partial u}\right)+\frac{\partial}{\partial u}\left(\frac{h_u\partial U}{h_v\partial v}\right)\right]-\frac{\mu\sigma_s}{d}\frac{\partial U}{\partial t}=\sigma_s\frac{\partial B_n^{ext}}{\partial t} \qquad (8)$$

with the initial and the boundary conditions

$$U(u, v, t) = 0,\ F(u, v, U_t, U_u, U_v, t) = 0. \qquad (9)$$

Considering the following input data: $d = 10^{-3}$ m, $\mu = 4\pi 10^{-7}$ H/m, $\sigma_V = 10^7$ 1/Ω/m, $\sigma_S = 10^4$ 1/Ω, for a toroidal wall geometry with holes, the function $U(x, y, t)$ at different time, excited by an $m=n$ =3/2 external kink mode in a thin wall with holes is presented in Fig. 2 and Fig. 3.

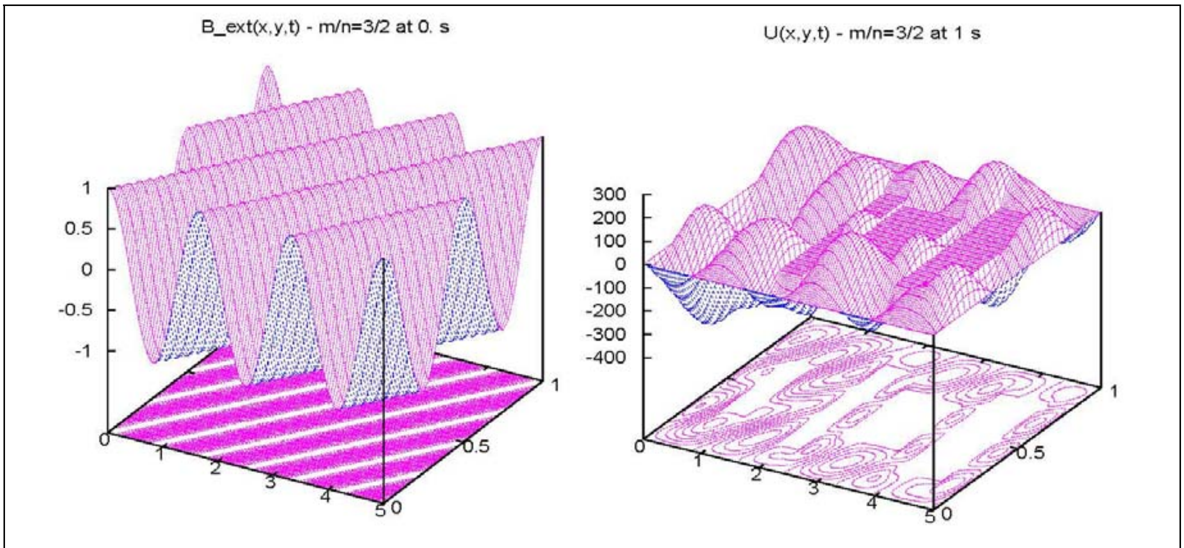

Figure 2. Perturbed magnetic field and stream function U of the induced eddy currents given by an EKM.

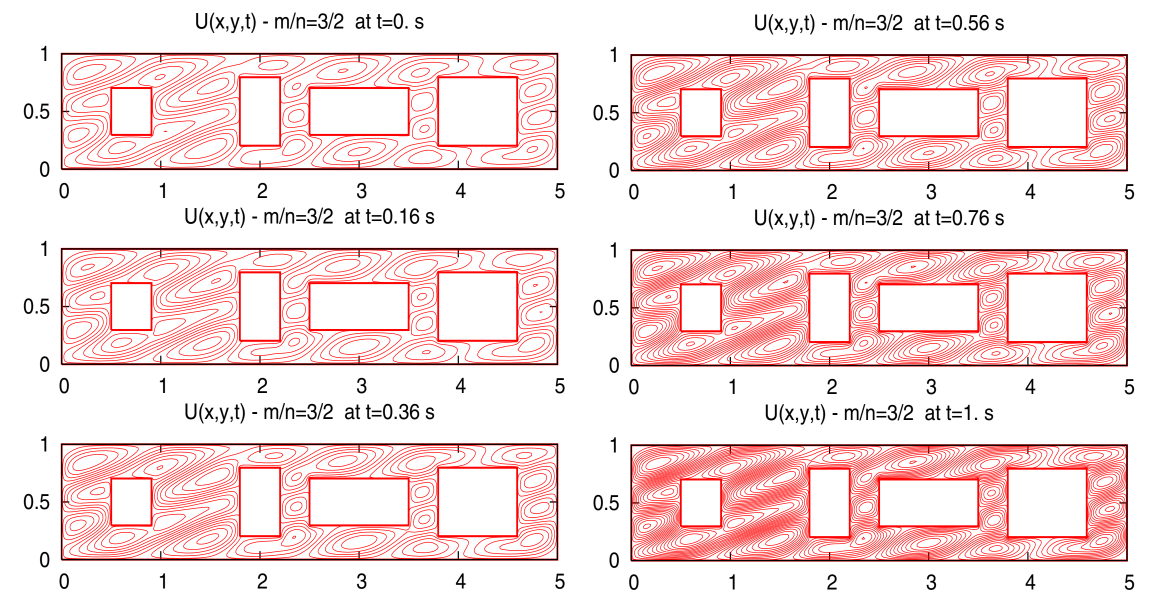

Figure 3. The eddy current stream function U(x, y, t) at different time, excited by an m/n =3/2 external kink mode in a thin wall with arbitrary holes.

Numerical treatment of holes singularities in time dependent problems

Special attention has been given to the accurate calculation of the influence of the eddy currents on the boundary conditions of the system of equations describing the RWM. If for an elliptical type problem, this is a "classical" task, this is not for a parabolic one.

We have developed a method for treating singularities which occur in solutions of parabolic partial differential equations due to sharp corners in the boundary. This method is used in conjunction with the simple explicit finite-difference scheme and subsequently the overall method is explicit. The standard finite-difference in such a neighbourhood was replaced by a truncated series representation of the exact solution at points close to the corner. The coefficients of this truncated series are estimated at each time step in terms of the solution values at points where the influence of the singularity is neglected and which have been derived by an explicit finite-difference scheme from the previous time step.

On the left hand side of Fig. 4, the dependence of the relative error of the numerical solution V_n with respect to the analytical one V_a for different numbers of discrete meshes of the domain N are given. On the right hand side of the same figure, the relative error with removal of the corners singularities is reported for the same numbers of discrete meshes.

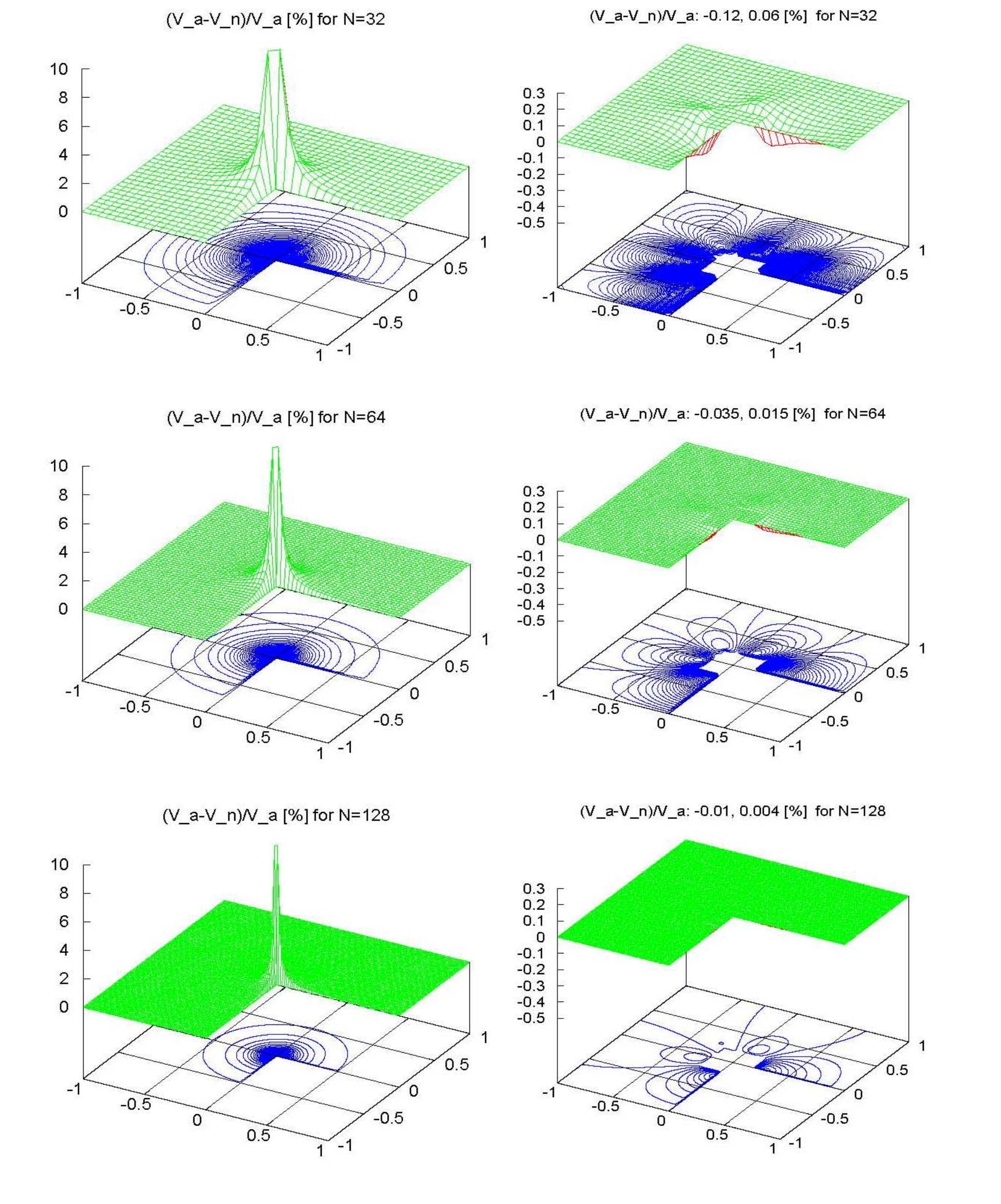

Figure 4. On the left hand side, the relative errors distribution around a re-entry corner for different numbers of discrete meshes with respect to the analytical solution without correction. On the right hand side, the relative errors distribution around a re-entry corner for the same numbers of discrete meshes but with analytically removed singularity.

This work has been carried out in close collaboration with our German colleagues from the Tokamak Physics Department of the Max-Planck-Institut für Plasmaphysik, during the mobility 27.07.07-05.10.07 at IPP Garching and at our home institute NILPRP. Part of this work has been performed in cooperation with Dr. L.E.Zakharov from PPPL, Princeton.

Next steps:

- to introduce, according to our semi-analytical model, feedback coils and detector sensors;

- to consider a toroidal wall with gaps and holes, compatible with the geometry of the ASDEX Upgrade and JET tokamaks;
- to continue the investigation of different dissipation mechanisms in the plasma.

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