STATISTICAL PHYSICS FOR ANOMALOUS TRANSPORT IN PLASMAS

Florin Spineanu, Madalina Vlad

Plasma Theory Group,
National Institute of Laser, Plasma and Radiation Physics, Magurele

1. Overview

The main research topics for this year comprise two objectives: the development of a model for the enhanced plasma resistivity due to generation of entanglement of the magnetic flux tubes and the study of nonlinear effects of the ExB drift in turbulent tokamak plasmas. Our research program was focused in 2007 on five milestones with strong theoretical and experimental relevance. We present below the two objectives with their milestones and the main results. The detailed description of the results is presented in Section 2.

Objective 1: ***Model for the enhanced plasma resistivity due to generation of entanglement of the magnetic flux tubes***

Milestone 1

Analytical method for the evaluation of the linking number generated by local fluctuation of the magnetic helicity.

Milestone 2

Calculation (in the drift-kinetic framework) of the enhanced resistivity in the magnetic stochastic regions by evaluating the effect of curvature related to the topological linking of the magnetic lines.

This research is a development of the original ideas proposed by the Plasma Theory Group for the quantitative description of the constraints imposed by the topology of the magnetic field configuration to the phenomenon of magnetic reconnection.

The main results

1. We have described the topological constraints on the stochastic magnetic configuration when a transient increase of helicity occurs in a finite plasma volume. Via bounds related to the magnetic energy that can be stored in that volume (*i.e.* statistical stationarity can be attained) a scaling was derived between the helicity and the average curvature of a generic magnetic line in the volume.

2. The particles' curvature-drift induces a new dissipation. We have estimated the contribution of this mechanism to the resistivity and we have shown that it is not substantial. However the new instruments that imply the topology of magnetic field are useful for quantifying the trapping of a magnetic line and for determining the reduction of the classical magnetic diffusion.

Objective 2: ***Nonlinear effects of the ExB drift in turbulent tokamak plasmas***

Milestone 1

Effect of trajectory trapping on density and impurity peaking.

Milestone 2

Analytical method for evaluation of the transport coefficients of test particles in electromagnetic turbulence.

Milestone 3

Assessment of the effect of trajectory trapping on structure generation in turbulent confined plasmas and calculation of the nonlinear growth rate of drift modes.

We have shown that the ExB stochastic drift produces strong non-linear effects in the regime characterized by trajectory trapping. There is memory in the motion, which determines a class of anomalous diffusion regimes. Trajectory quasi-coherent structures are produced. An average velocity is produced in turbulent plasmas through a ratchet-type process due to the space variation of the confining magnetic field [1]. These effects have been found by means of the semi-analytical statistical methods that we have developed (*the decorrelation trajectory method* [2] and *the nested subensemble approach* [3]*).*

These results were developed in three directions contained in the milestones of this objective.

The main results

1. We have developed the model for the ratchet pinch in turbulent magnetically confined plasmas by introducing particle collisions and an average velocity V_d that describes plasma poloidal rotation. Strong effects on the ratchet velocity and on the peaking factor were shown to appear. Large peaking factors (of the order of the experimental ones) for impurities and for plasma density are produced only through nonlinear effects at large Kubo numbers and in the presence of a weak poloidal rotation with a velocity smaller than the amplitude of the stochastic ExB velocity.

2. The decorrelation trajectory method was extended to the study of test particle transport in electromagnetic turbulence. We have obtained strong effects on both electrons and ions in the nonlinear regime. The magnetic component of the field has a decorrelation effect for the ions that leads to the increase of the diffusion coefficient. The effect on the electrons is to reduce the transport coefficient due to the stochastic magnetic islands that appear and trap the electrons.

3. The connection between the trajectory trapping and the structure generation in confined plasmas was analysed. We have shown that the ExB drift in turbulent plasma determines trajectories that have a high degree of coherence and long time memory. They form transitory structures with characteristics that depend on the turbulence. We have determined the effect of these structures on the growth rate of drift modes on turbulent plasma and we have shown that the evolution to large scale is produced by the ion trajectory structures. A different perspective on the inverse cascade is obtained. It does not appear as wave-wave interaction but as the effect of ion ExB motion on the drift wave stability.

These results are presented in 8 publications [P1]-[P8].

2. <u>Results obtained in 2007</u>

2.1. Model for the enhanced plasma resistivity due to generation of entanglement of the magnetic flux tubes

The energy of the stochastic magnetic field is bounded from below by a topological quantity expressing the degree of linkage of the field lines. When the bound is saturated one can assume that the storage of a certain magnetic energy requires a minimal degree of topological complexity. It is then possible to infer a connection between the helicity content and the average curvature of the magnetic field lines. The random curvature induces random drifts leading to an additional dissipation and modified resistivity [P1]. Extensions to statistical properties of linking were analysed.

When the Chirikov criterion is verified for several chains of magnetic islands (developing at closely neighbour resonant magnetic surfaces in a volume of plasma) the magnetic field becomes stochastic. In general the magnetic stochasticity is taken into account in transport processes due to the high efficiency of energy spreading through the stochastic region. However from the point of view of the structure of the magnetic field it is difficult to say anything more than we know from Hamiltonian chaotic systems: there is a stochastic (exponential) instability, local Lyapunov exponents and Kolmogorov length and the test particles move diffusively or have various sub- and supra-diffusive behaviours.

However in a stochastic region the field must still obey some constraints. These constraints arise from the relation between the energy stored in the magnetic field and the topological complexity of the field. The constraints can be briefly expressed in this way: it is not possible to support a certain energy in a volume spanned by transiently stochastic magnetic field lines if these magnetic field lines do not have a certain minimal degree of topological complexity.

This should be seen in relation with the equation that expresses the topological *link* in terms of *writhe* and *twist* and in relation with the dynamics of a twisted flux tube. If an initial amount of link is stored exclusively as twist, then beyond a certain level of twist the flux tube deforms and acquires writhe, thus distributing the higher amount of link into the two kinds of topological deformations: twist and writhe. In plasma free from strong magnetic background (as in astrophysical plasma or solar corona) generation of writhe means a coiling or super-coiling instability, a large spatial deformation. In a tokamak the stochastic flux tubes are also subject to the writhing instability when a local fluctuation of the parallel electric field occurs, but they are more constraint by the confining $\mathbf{B}_0$ and cannot perform large spatial displacements. Instead, as a result of small deformations originating from local writhing (coiling) instability, they will reconnect such that, from elements of tubes, effectively new strings are created, with a new effective entanglement. It is reasonable to assume that these new, episodic, flux tubes, by their mutual linking, satisfy on the average the energy-topology constraints. Therefore we assume that the field flux tubes inside the stochastic region reconnect to generate transiently configurations that exhibit a certain topological entanglement. Together with the dynamical nature of the stochasticity phenomena, the formation of these entangled structures is transient and we may suppose that the higher topological content results from a statistical average. At large time the topological reduction occurs with suppression of relative linking via tube merging, a process called by Parker *topological dissipation*.

The curvature of magnetic flux tubes induces drifts of particles. Electrons and ions flowing along curved magnetic lines will have opposite drifts and local charge separations produce random transversal electric fields. For a finite collisionality this is a source of additional dissipation. We have estimated the negative current perturbation due to curvature.

2.2 Nonlinear effects of the ExB drift in turbulent tokamak plasmas

2.2.1. Effect of trajectory trapping on density and impurity peaking

We have developed an alternative model for particle pinch in tokamak plasma based on test particle approach. We have shown that an average velocity is produced in turbulent plasmas through a ratchet-type process due to the space variation of the confining magnetic field [1]. We have shown that the inhomogeneity of the confining magnetic field determines a directed transport (an average velocity), although the average ExB drift velocity is zero.

We have developed this model for the direct transport in turbulent magnetically confined plasmas by introducing particle collisions and an average velocity V_d that describes plasma poloidal rotation. These two processes are always present in plasmas and thus it is important to know their effects. Studies of the transport for constant magnetic field have shown that both collisions and average velocity have very strong effects and determine anomalous diffusion coefficients, which appear in a turbulence with K>1 in the condition when particle eddying motion persists. The average velocity and the diffusion coefficients were determined using our semi-analytical statistical approach, *the decorrelation trajectory method*. The combined action of both perturbations was determined. Strong effects on the ratchet velocity and on the peaking factor were shown to appear. We have shown that the collisional displacements lead to a correlation between the stochastic potential and the inhomogeneity of the magnetic field, which increases the effects of collisions on the ratchet pinch. It also determines the change of the direction of the ratchet pinch at large Kubo numbers. This work is submitted for publication [P2].

The ratchet pinch appears in the frame of test particle approach and thus we determine the effects of the non-homogeneity of the magnetic field on particle trajectories. We note that additional effects appear when a density of particles is considered: the divergence of the velocity determines the modulation of the density along trajectories and the concentration in the regions where the divergence of the velocity is negative. This density concentration produces a pinch velocity, the curvature or turbulent equipartition pinch. This effect does not appear in the test particle approach. We have studied the relation between these two pinches and between test particle and scalar field advection approaches. We have determined the average velocity produced by the gradient of the magnetic field on the density and we have shown that it is a combination of the ratchet and curvature pinch. We have determined the effect of ratchet pinch on the density (passive scalar) and we have extended the studies of passive scalar transport to the nonlinear regime. The preliminary results were presented at “Festival de Theorie”, Aix-en-Provence [P3] and at the 49th Annual Division of Plasma Physics Meeting [P4].

2.2.2. Analytical method for evaluation of the transport coefficients of test particles in electromagnetic turbulence

We have studied in previous work test particle transport both in electrostatic and magnetic turbulence. The decorrelation trajectory method was extended to the study of test particle transport in electromagnetic turbulence. A numerical code was developed, which determines the diffusion coefficients for electrons and ions for given spectrum of the electromagnetic field. The results show strong effects on both electrons and ions in the nonlinear regime. The magnetic component of the field has a decorrelation effect for the ions that leads to the increase of the diffusion coefficient. The effect on the electrons is the decrease of the diffusion coefficient. This is produced due to the stochastic magnetic islands that appear at large magnetic Kubo numbers and consists in solenoidal segments of the magnetic lines. The magnetic islands have a trapping effect that determines the decrease of the diffusion coefficient. The perturbed magnetic lines also determine for the electrons an effective parallel decorrelation time that depends on the magnetic Kubo number.

We have developed in collaboration with Université de Provence, Marseille a model of Hasegawa-Wakatami type for the electromagnetic turbulence in non-homogeneous magnetic field. A computer code for the simulation of the turbulence is developed. Detailed comparison of the numerical and analytical results will contribute to better understanding these nonlinear effects on the transport.

2.2.3. Assessment of the effect of trajectory trapping on structure generation in turbulent confined plasmas and calculation of the nonlinear growth rate of drift modes

Detailed statistical information about particle trajectories was obtained using the nested subensemble method [3]. This method determines the statistics of the trajectories that start in points with given values of the potential. This permits to evidence the high degree of coherence of the trapped trajectories. Their average displacement, dispersion and probability distribution function saturate in a time τ_s. The time evolution of the square distance between two trajectories is very slow showing that neighbouring particles have a coherent motion for a long time, much longer than τ_s. They are characterized by a strong clump effect with the increase of the average square distance that is slower than the Richardson law. These trajectories form structures, which are similar with fluid vortices and represent eddying regions. The statistical parameters of these structures (size, build-up time, dispersion) were determined. In time dependent potentials the structures with $\tau_s > \tau_c$ are destroyed and the corresponding trajectories contribute to the diffusion process. The average size of the structures S in a time dependent potential was determined as function of the Kubo number K. For K<1 the structures are absent (S~0) and they appear for K>1 and continuously grow as K increases.

These results obtained for the statistics of test particle trajectories were used for the study of test modes on turbulent plasmas by developing a Lagrangian approach that takes into account trajectory trapping. Studies of plasma turbulence based on trajectories were initiated by Dupree and developed especially in the seventies. These methods do not account for trajectory trapping and thus they apply to the quasilinear regime or to unmagnetized plasmas. A very important problem that has to be understood is the effect of this non-standard statistical behaviour of the test particle trajectories on the evolution of the instabilities and of turbulence in magnetized plasmas. We have studied linear test modes on turbulent plasma for the drift instability in slab geometry with constant magnetic field. The combined effect of the parallel motion of electrons (non-adiabatic response) and finite Larmor radius of the ions destabilizes the drift waves. We consider a turbulent state of the plasma with known statistical characteristics of the electrostatic potential. The perturbations of the electron and ion distribution functions are obtained from the gyro kinetic equation as integrals along test particle trajectories of the source terms determined by the density gradient.

The background turbulence produces two modifications of the equation for the linear modes. One consists in the stochastic ExB drift that appears in the trajectories and the other is the fluctuation of the diamagnetic velocity. Both effects are important for ions while the response of the electrons is approximately the same as in quiescent plasma. The average propagator of the modes is evaluated using the above results on trajectory statistics. In the first order it depends on the size S(K) of the structures. The trajectory trapping process has a complex influence on the mode. The ion trajectory structures (the quasi-coherent component of their motion) determine the S-dependent exponential factor in the frequency ω. Its effect is the displacement of the unstable k-range toward small values. The random component in the ion motion determines a diffusive damping term in the growth rate γ that produces the stabilization of the large wave numbers. The fluctuations of the diamagnetic velocity determine the last term in the growth rate. The tensor R_{ij} contributes to the growth of the modes.

A different perspective on the inverse cascade is obtained. It does not appear as wave-wave interaction but as the effect of ion ExB motion on the drift wave stability. Namely, the quasi-coherent motion of the trapped ions produces the destabilization of the modes with wavelengths of the order of the average size of the trajectory structures. This decreases the frequency of the modes and produces the increase of the size S. The non-standard statistical properties of the trajectories, namely the formation of trajectory quasi-coherent structures, are shown to be associated with order and structure formation in turbulent magnetized plasmas. These results are presented in [P5] and [P6].

3. Collaborative actions

3.1 Collaboration with EFDA JET

We have continued the collaboration with EFDA JET included in the Transport Task Force initiated last year. The main topics and results are presented below.

T-3.3.2. Determination of parametric dependences of ITG and TEM thresholds

- We have participated to the identification of physical conditions appropriate for the experimental study of the threshold for the onset of Ion Temperature Gradient driven turbulence. In particular, the examination of the role of poloidal rotation in the threshold value. Considering the fast variation of the linear growth beyond the threshold we have started an analytical study of the effect of fluctuations (small scale turbulence) on the precision with which the threshold can be identified. The effect of plasma rotation has been included. A paper is in preparation.

- Evaluation of the free-energy source available for the density pinch in the stationary regime. We have derived the general expression of the energy for a stationary vortex in two-dimensional plasma [P7, P8]. We proved that there are neighbouring states of lower energy, which are accessible if the system is weakly driven by vorticity input. We have proved that a minimum energy is reached when the vortex is quasi-singular, with all vorticity concentrated on the magnetic axis. This provides a new contribution to the density pinch, independent of turbulence.

T-3.3.3. Study of impurity transport and control

- We have participated at the experiments on the study of impurity transport and control using He3.

- We have developed our pinch model based on a ratchet type process, which includes particle collisions and poloidal average flows, by taking into account the mass-charge dependence of the collisional diffusivity and its anisotropy. The anisotropy is determined by the Larmor radius dependence on the R dependent toroidal magnetic field. A new program based on the decorrelation trajectory method was developed and tested. An interesting effect was obtained that consists in the change of the sign of the pinch velocity from inward to outward at large Kubo numbers.

- A series of calculations for the range of parameters that are of interest for JET plasmas provide the following preliminary results:

\- The ratchet pinch is compatible with the Z dependence obtained in experiments, including the change of sign.

\- The absolute values of the peaking factor are smaller due to the effective diffusion that has values larger than in experiments.

After the Campaigns, this work was continued by improving the treatment of collisions in the frame of the decorrelation trajectory method. A correlation between the stochastic potential and the space dependent magnetic field was shown to appear due to collisions. This determines a stronger effect of collisions on the pinch velocity. Also, the relation between the ratchet pinch and the curvature pinch, both determined by the gradient of the magnetic field, was understood. These theoretical results are presented in two papers, one is submitted for publication and the other is in preparation.

Secondments:

- Dr. F. Spineanu at JET for 28 days *(22/01/2007 – 16/02 /2007).*
- Dr. M. Vlad at JET for 28 days *(22/01/2007 – 16/02 /2007).*

3.2 Participation at ITM-Project 4: Microinstabilities and Turbulence.

We have proposed the study of the following topics in the ITM-Project 4.

1. F**ast particle turbulent transport**

We have found that, in some conditions, turbulence induced transport at the very large Larmor radii corresponding to fusion produced particles can be important. This happens when the turbulence has slow time variation that corresponds to large Kubo numbers. We think that this problem is very important for ITER and we have discussed it with several participants to TF T and ITM-Project 4. We have begun a series of studies with the aim of better understanding this fundamental process. We have studied the effect of the poloidal rotation of plasma on fast particle turbulent transport. A code was developed based on decorrelation trajectory method that includes the parallel motion, and preliminary results were obtained.

2. **Density peaking induced by the dynamics of the vorticity**

We have developed the idea (which is proposed for collaborative work) that the **self-organization of the vorticity** of the background equilibrium plasma is as important as turbulence (which has captured most of the present-day preoccupations). Density pinch, momentum transport, impurity accumulation or expelling and profile resiliency are signatures of the fact that the background plasma reaches a quasi-coherent state, with a balance of drive and turbulent transport [8], [9]. We have derived a differential equation that should describe the vorticity equilibria.

The experimental verification is proposed, consisting of observation of the large scale (not turbulent) vorticity dynamics in various conditions.

3.3 Collaboration with CEA Cadarache and Universite de Provence

3.3.1. Effect of trajectory trapping on density and impurity peaking

We have continued the collaboration on this topic, which started in the year 2005. We have developed our model for average velocities produced in turbulent plasma, the ratchet pinch.

The main results of this collaboration are presented in Section 2.1.

Mobility Secondment:

- Dr. M. Vlad at Universite de Provence – CEA Cadarache for 68 days (4/05-10/07/2007)

3.3.2. Calculation of the enhanced resistivity due to the higher topological content of the stochastic magnetic fields in tokamak plasmas

We have developed explicit analytical procedures for the evaluation of the effect of the topological properties of the field, based on the concept of invariant magnetic helicity. This allows the investigation of the bound in the energy of the magnetic field connected with the total topological link in the plasma volume. We have estimated the amount of link that is generated when a quantity of energy is injected into a volume of plasma. Connecting the geometry of a line intersecting a surface and the link between that line with respect to the board of the surface we have estimated the curvature of the line and its relation with the total helicity. The difference between the electron and ion curvature drifts represents a current and actually induces an effective turbulent resistivity which we have calculated.

Mobility Secondment:

- F. Spineanu at Universite de Provence – CEA Cadarache for 87 days (3/10-30/12/2007)

References

[1] **M. Vlad**, **F. Spineanu**, **S. Benkadda**, "*Impurity pinch from a ratchet process*", Phys. Rev. Letters 96 (2006) 085001.

[2] **M. Vlad, F. Spineanu, J.H. Misguich, R. Balescu**, Phys. Rev. E 58 (1998) 7359.

[3] **M. Vlad, F. Spineanu**, *"Trajectory structures and transport"*, Phys. Rev. E 70 (2004) 056304.

List of publications in 2007

[P1] F. Spineanu, M. Vlad, "*Helicity fluctuations, generation of linking number and effects on resistivity*", **International Review of Physics (IREPHY) 1** (2007) 65.

[P2] M. Vlad, F. Spineanu, S. Benkadda, "*Collisions and average velocity effects on ratchet pinch"*, **Physics of Plasmas**, submitted 2007.

[P3] M. Vlad, F. Spineanu, S. Benkadda, "*Ratchet versus curvature pinch*", Festival de Theorie, Aix-en-Provence, 2007.

[P4] M. Vlad, F. Spineanu, S. Benkadda, *„Ratchet and curvature pinch in turbulent plasmas",* 49th Annual Division of Plasma Physics Meeting, November 2007, Orlando, Florida.

[P5] M. Vlad, F. Spineanu, *"Trajectory trapping and structure generation in turbulent magnetized plasmas",* 49th Annual Division of Plasma Physics Meeting, November 2007, Orlando, Florida, oral presentation.

[P6] M. Vlad, F. Spineanu, *"Test particles and test modes in plasma turbulence",* Annals of the University of Craiova, special issue in honor of R. Balescu (2007).

[P7] F. Spineanu, M. Vlad, S. Benkadda, *„The basic evolution of the angular momentum density in a field-theoretical model of vorticity transport",* 49th Annual Division of Plasma Physics Meeting, November 2007, Orlando, Florida.

[P8] F. Spineanu, M. Vlad, *"The large scale two-dimensional stationary vortex in magnetized plasma"* Annals of the University of Craiova, special issue in honor of R. Balescu (2007).