I Physics of the Tokamak plasma

INTERPRETATION AND CONTROL OF HELICAL PERTURBATIONS IN TOKAMAKS

C.V. Atanasiu, A. Moraru

Mathematical Modelling for Fusion Plasmas Group

National Institute for Lasers, Plasma and Radiation Physics, Magurele

1 Overview

During the period January-December 2009, the theoretical and modelling research activity of the "*Mathematical Modelling for Fusion Plasmas Group*" of the National Institute for Lasers, Plasma and Radiation Physics (NILPRP), Magurele - Bucharest, Romania has been focalized on:

Resistive wall modes stabilization, activity performed in collaboration with the Max-Planck - Institut für Plasmaphysik (IPP), Tokamakphysics Department, Garching, Germany, with the following **specific objectives:**

- ***Optimisation in the calculation of the wall response (for a real 3D geometry, with holes) to external kink mode perturbations;***
- ***Investigation of different edge dissipation mechanisms with our semi-analytical RWM model and code***

The **objective** of this research made in common with IPP Garching, Tokamakphysics Department, under the frame of ITM and TG-MHD task forces, is to advance the physics understanding of RWMs stability, including the dependence on plasma rotation, wall/plasma distance, and active feedback control, with the ultimate goal of achieving sustained operation at beta values close to the ideal-wall beta limit through passive and active stabilization of the RWMs.

2 Results

2.1 Optimisation in the calculation of the wall response (for a real 3D geometry, with holes) to external kink mode perturbations

a) Fixed plasma with respect to the wall case

We have continued our approach of calculating the wall response to an external kink mode perturbation with the help of a scalar potential (current stream function) [1- 5]. We have shown that such approach, necessitating the calculation of the unknown function over the wall volume only, is more convenient than other approaches using a vector potential, to be determined in the whole space. Such point of view has been presented and accepted at the ITM General Meeting in Frascati, August 2008.

A new curvilinear coordinate system *(u,v,w)*, more suitable to describe a tokamak wall, has been defined, where two covariant basis vectors $(\boldsymbol{r}_u, \boldsymbol{r}_v)$ are tangential to the wall surface and the third vector $\boldsymbol{r}_w$ is normal to the wall surface, the following partial differential equation, describing the time evolution of the induced surface currents (i.e. the wall response to the external kink mode) has been written

$$\frac{\partial(\vec{n}\bullet\vec{B})}{\partial t} = d\,\frac{\partial(\vec{r}^{\,h}\bullet\vec{B})}{\partial t} = \frac{1}{D}\left[\frac{\partial}{\partial u}\frac{1}{\sigma d}\left(\frac{g_{vv}}{D}\frac{\partial V}{\partial u} - \frac{g_{uv}}{D}\frac{\partial V}{\partial v}\right) + \frac{\partial}{\partial v}\frac{1}{\sigma d}\left(\frac{g_{uu}}{D}\frac{\partial V}{\partial v} - \frac{g_{uv}}{D}\frac{\partial V}{\partial u}\right)\right]$$

where d is the thickness of the wall and V is the scalar potential of the surface currents, defined such as

$$\vec{i} = \nabla V \times \vec{n}_s;\ \nabla\bullet\vec{i} = 0.$$

with **I** surface current density and $\mathbf{n}_s$ the external normal to the wall. σ is the electric conductivity of the wall. The metric tensor has four components only, g_{uu}, g_{vv}, g_{uv} and $g_{ww}=d^2$. D is the 2D Jacobian at the wall surface: $D=g_{uu}\cdot g_{vv}-(g_{uv})^2$.
The magnetic field contains two components: $\boldsymbol{B}^{pl}$ – the exciting component due to the external kink modes of the plasma- and $\boldsymbol{B}^{eddy}$ – the field created by the eddy currents them self. Both have been taken into account in our calculations.

Another considered model, describing the wall response to an external kink mode, is based on the formulation with the help of the magnetic vector potential $\boldsymbol{A}=\boldsymbol{A}^{pl}+\boldsymbol{A}^{eddy}$ and leads to the following integro-differential equation, with the magnetic vector potential $\boldsymbol{A}$ and the scalar potential of the surface currents V as unknowns

$$\nabla V(\vec{r})\times\vec{n}_s(\vec{r}) + \frac{\mu_0\sigma(\vec{r})d}{4\pi}\int_S \frac{\partial\left[\nabla V(\vec{r}\,')\times\vec{n}_s(\vec{r}\,')\right]/\partial t}{\sigma(\vec{r}\,')d\,|\vec{r}-\vec{r}\,'|}dS' = \sigma(\vec{r})\left[\frac{\partial\vec{A}^{ext}}{\partial t} + \nabla U_e(\vec{r})\right].$$

U_e is the electric scalar potential to be determined from a Laplace type equation with Neumann boundary conditions. S is the wall surface. This model is now under our investigation.

We have elaborated a new methodology to solve the PDE. In Table 1, the running time obtained with the new method appeared to be with at lest one order of magnitude faster than a classical solving approach.

To our knowledge, it is the first time that such a method, applied to the numerical solving of PDE, having the domain of application a curvilinear complex geometry, has been reported.

Solving method	**No. of grid points**	**Running time [s]**
classical	101 x 101	103
new	101 x 101	3
classical	151 x 151	690

new	151 x 151	14

Table 1. *Comparison of running times between the classical solving method and the new method developed by us.* 101x101 *means:* 101 *grid intervals on "u" direction,* 101 *grid intervals on "v" direction.*

To check the accuracy of our method, we have used Stokes theorem by performing some line integrals and verifying the corresponding surface integrals. At sufficiently high number of grid points, an excellent agreement between both integrals has been found – overlapping up to the 6th significant digit. Besides this checking, we have imagined some simple cases permitting an analytical solution. The results of this accuracy test are given in Table 2.

METHOD	**No. of grid points/order of approx. $O(h^n)$**	**Contour Integral around the wall**	**Contour integral around a hole**	**Scalar potential of the surface current** V
Analytical		-64.93939402267	-1753.363638612	-88.57368818753
Numerical	51x51 / 1	-64.93939402252	-1727.830192664	-88.10910495617
	51x51 / 2	-64.93939402246	-1753.163995570	-88.58057153492
	51x51 / 3	-64.93939402296	-1753.297066133	-88.56891335580
	101x101 / 1	-64.93938644007	-1740.487312817	-88.34391781803
	101x101 / 2	-64.93938664805	-1753.312709730	-88.57543866452
	101x101 / 3	-64.93939488308	-1753.346663857	-88.57247069322
	151x151 / 1	-64.93642505521	-1744.751658112	-88.42056389921
	151x151 / 3	-64.93912067249	-1753.340452717	-88.57441239957
	151x151 / 3	-64.93940394442	-1753.355924635	-88.57313333700

Table 2. *Accuracy test with respect to an analytical case.* 101x101 / 3 *means:* 101 *grid intervals on "u" direction,* 101 *grid intervals on " v" direction,* / 3 = $O(h^3)$

b) Rotating plasma case

With the assumption that the plasma cross-section perpendicular to the moving direction is constant, by using the Minkowski formulation for the Maxwell equations and Ohm's law, the following diffusion like equation has been obtained

$$\frac{\partial(\vec{B}^{eddy})}{\partial t} = \frac{1}{\mu\sigma}\nabla^2\vec{B}^{eddy} + [(\vec{B}^{pl} + \vec{B}^{eddy})\bullet\nabla]\vec{v} - (\vec{v}\bullet\nabla)(\vec{B}^{pl} + \vec{B}^{eddy}) - \frac{\partial\vec{B}^{pl}}{\partial t}.$$

where $\mathbf{B}^{pl}$ is the perturbed magnetic field created by the external kink mode of the plasma, and

$$\frac{\vec{v}}{r^{pl}} = m\Omega_\theta - n\Omega_\zeta,\ \Omega_\theta = \frac{d\theta}{dt},\ \Omega_\zeta = \frac{d\zeta}{dt}$$

r^{pl} is some plasma minor radius, and θ and ζ are the poloidal and toroidal angles, respectively.

Next steps:

- to finalize the rotating plasma case;

- to include in our model the interaction of the plasma and wall with the feedback and error coils, by using our concept of surface current [6, 7].

1.2.3 Investigation of different edge dissipation mechanisms with our semi-analytical RWM model and code

We have considered the following edge dissipation mechanisms: (1) due to the anomalous plasma viscosity (2) due to charge-exchange with cold neutrals, (3) due to neoclassical flow-damping, (4) due to sound-wave damping.

For this task, we have continued to consider the seminal Fitzpatrick plasma model [8], with the standard large-aspect ratio, low β, circular cross-section tokamak plasma [9, 10, 11], we have drawn the linearized ideal MHD equations, by considering the following perturbed quantities: magnetic field, current density, plasma velocity, plasma pressure, plasma parallel stress tensor, polarization current and the neoclassical current [12, 13]. The perturbed quantities have to be in the frame of the standard assumptions in single-mode neoclassical theory. For the sake of simplicity, the plasma equilibrium has been considered as force-free (i.e., zero pressure gradient and zero diamagnetic current)

$$\vec{J} = \varepsilon_0 \frac{g}{B}\vec{B},\ \varepsilon_0 = a/R_0,\ g(r) = \frac{1}{r}\frac{d}{dr}\left(\frac{r^2}{q}\right)$$

the magnetic field is expressed in second order approximation of the aspect ratio and as safety profile the well known Wesson profile [14] has been used. Even if really, the ideal external kink modes are driven by plasma pressure gradients, in this model, for sake of simplicity, these modes have been considered driven by current gradients.

The following set of linearized ideal MHD equations have been used

$$\gamma'\tilde{\vec{b}} = \nabla\times(\tilde{\vec{v}}\times\vec{B}),\ (\gamma' + \upsilon)\rho\tilde{\vec{v}} = -\nabla\tilde{p} - \nabla\cdot\tilde{\vec{\Pi}}_{\parallel} + \tilde{\vec{j}}\times\vec{B} + \vec{J}\times\tilde{\vec{b}},$$

$$\gamma'\tilde{p} = -\rho c_s^2\nabla\cdot\tilde{\vec{v}}.$$

($\tilde{\vec{\Pi}}_{\parallel}$ is the perturbed plasma parallel stress tensor, $\gamma' = \gamma + \mathrm{i}n\Omega_{\phi}$ is the mode growth rate in the plasma frame, with n the toroidal wave number and Ω_{ϕ} the toroidal angular plasma velocity, $c_s(r)$ the plasma sound speed, $\upsilon(r)$ a plasma flow damping rate).

The stability parameter considered in the following will be

$$\kappa = \frac{\beta_c - \beta_{nw}}{\beta_{pw} - \beta_{nw}}$$

where β_{nw} is the no-wall beta limit, β_{pw} is the perfect wall beta limit, while β_c is the current beta (if κ=1, then the RWM is stabilized, if κ=0 the RWM is not stabilized).

In Fig. 1, the RWMs stability boundaries in function of the toroidal plasma rotation for different plasma flow damping rates (due to charge exchange with neutrals) are given, while in

Fig. 2 the RWMs stability boundaries in function of the toroidal plasma rotation for different plasma neoclassical flow damping parameters are presented.

The dissipation due to the sound wave damping (at the edge) is negligible as stabilization effect.

The RWMs stability boundaries in function of the toroidal plasma rotation for different plasma edge perpendicular viscosities are reported in Fig. 3.

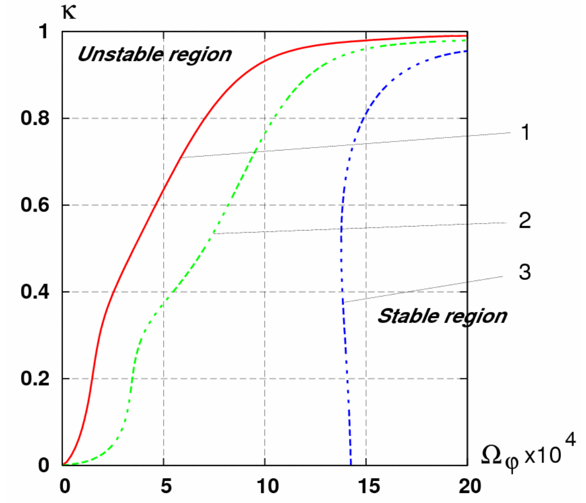

Fig. 1 Stability parameter κ as function of the toroidal plasma rotation Ω_Φ, for different plasma flow damping rates υ due to charge exchange with neutrals: (1) $\upsilon = 7\times10^4$, (2) $\upsilon = 4\times10^4$, (3) $\upsilon = 1.7\times10^3$, m/n=3/1, q(a)=3. $\upsilon \approx v_i n_n \sigma_x$, $v_i = \sqrt{T_i / m_p}$ is the ion thermic velocity, n_n is the neutral density, σ_x is the charge exchange cross-section.

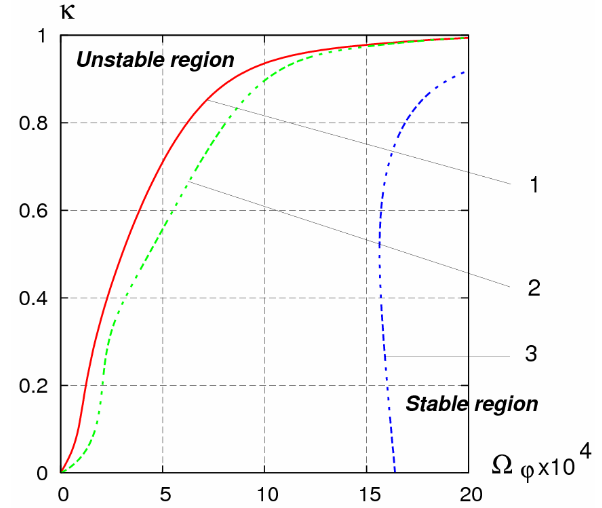

Fig. 2 Stability parameter κ as function of the toroidal plasma rotation Ω_Φ, for different neoclassical flow damping parameters $\mu_\perp$, (1) $\mu_\perp = 9\times10^{-15}$, (2) $\mu_\perp = 4.5\times10^{-15}$, (3) $\mu_\perp = 9\times10^{-14}$, with ρ the plasma density (mass) and η_0 the parallel ion viscosity.

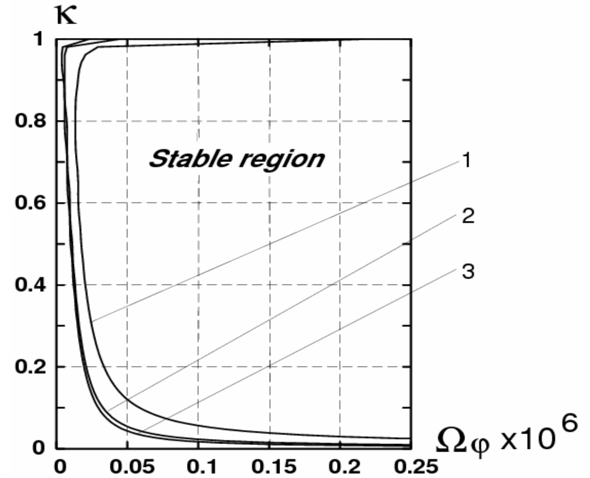

Fig. 3 Stability parameter κ as function of the toroidal plasma rotation Ω_Φ, for different plasma edge perpendicular viscosities $\mu_\perp$, (1) $\mu_\perp = 9\times10^{-15}$, (2) $\mu_\perp = 4.5\times10^{-15}$, (3) $\mu_\perp = 9\times10^{-14}$

Conclusions:

- *by using a model with phenomenological damping parameters, there is no evident what kind of dissipation mechanism really is taking place. A numerical simulation will be necessary;*

- *after running scenarios for different dissipation mechanisms and with a very large scale of plasma parameters, we have to accept that the results obtained with our model, based on Fitzpatrick's model do not correspond to experimentally realistic timescales and plasma rotation values (O(1%) Ω_A) even if it offers some useful information on the plasma rotation influence on mode growth rate. Presently, this seems to be a general opinion. With this in view, we intend to start the developing of a new more realistic analytical model with a resonant resistive-visco-inertial layer inside of the plasma.*

References:

[1] C.V. Atanasiu, S. Günter, A. Moraru, L.E. Zakharov, EPS34, Warsaw, Poland, 2-6 July 2007.
[2] C.V. Atanasiu, S. Günter, A. Moraru, L.E. Zakharov, EFTC12, Madrid, Spain, 24-27 September 2007.
[3] C.V. Atanasiu, A. Moraru, L.E. Zakharov, EPS35, Hersonissos, Crete, Greece, 9-13 June 2008.
[4] C.V. Atanasiu, A. Moraru, L.E. Zakharov, "*Response of a resistive wall with holes to an external kink mode in a tokamak*", 21st International Conference on Numerical Simulation of Plasmas, Lisbon, Portugal, 6-9 October 2009.
[5] C.V. Atanasiu, A. Moraru, L.E. Zakharov, "*Influence of a Nonuniform Resistive Wall on the RWM Stability in a Tokamak*", American Physical Society Plasma Meeting, Atlanta, USA, 2-6 November 2009.
[6] C.V. Atanasiu, A. Moraru, A. Subbotin, Computer Physics Communications 98 (1996).
[7] C. V. Atanasiu, S. Günter, K. Lackner, A. Moraru, L. E. Zakharov, and A. A. Subbotin, Phys. Plasmas **11**, 5580 (2004).
[8] R. Fitzpatrick, A.Y. Aydemir, Nuclear Fusion, **36**, 11 (1996).
[9] C.V. Atanasiu, *"Resistive wall modes stabilization in diverted tokamak configurations »*, invited paper, Hungarian Plasma Physics and Fusion Technology Workshop, 26-28 March 2008, Györ, Hungary.
[10] C.V. Atanasiu, Miron I.G., *"An analytical model for resistive wall modes stabilization"*, 11th European Fusion Theory Conference, 26-28 September 2005, Aix-en-Provence, France.
[11] C.V. Atanasiu, Miron I.G., "*A model for resistive wall mode control"*, EPS30, 7-11 July 2003, Sankt Petersburg, Russia.
[12] A. Bondeson, Ward, Phys D.J., Rev. Lett. **72**, 2709 (1994).
[13] G.W. Hammett, F.W. Perkins, Phys. Rev. Lett. **64**, 3019 (1990).
[14] J.A. Wesson, *Tokamaks*, (Oxford University Press, Oxford, UK, 2004).
[15] H. Zohm, J. Adamek, C. Angioni, G. Antar, C.V. Atanasiu, et. al, Nuclear Fusion **49** 104009 (9pp) (2009).