STABLE ORGANIZED MOTION, COHERENT STRUCTURES AND TURBULENCE IN TOKAMAK PLASMAS

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Overview

Two main subjects have been investigated :

- The organization of the vorticity field into cuasi-coherent structures at the edge of the tokamak plasma
- The filamentation process in the case of strong Edge Localised Modes

The aim of the first part of the work is to determine the conditions of obtaining a regime of parameters that allows to achive high confinement in tokamak plasma. Our approach is based on the idea that vorticity represents a self-ordered field. We study concrete problems of high interest for reactor plasmas: the peaking of density and respectively the fast drop of radial electric field at the edge of the plasma. These studies deal with several aspects of plasma evolution: generation of quasi-coherent structure by condensation at large scales, the energetic ground of the self-organization of the vorticity, threshold conditions separating distinct regimes of evolution of vorticity, density entrainment and relative functional extrema for stable vorticity profiles. Basically we use an original approach [1], which we have developed in the recent years and which has already produced effective results. This approach consists of formulating the vorticity field evolution as a field theoretical problem, based on a Lagrangian density. Numerical studies have been done with a high precision integration routine, since the equation is strongly nonlinear. This work is a contribution to the clarification of the role of coherent structures in reaching the H mode.

Dynamics of the vorticity under variational constraints and natural rotation profiles

A first objective for 2009 was to investigate the effect of density-vorticity common dynamics (Ertel's theorem) on the formation of a local maximum of density at the plasma edge in the H-mode. The aim is to understand the dynamics of H-modes and the generation of Edge Localized Modes (ELM's). The sheared rotation induces the suppresion of turbulent losses to a level that is very convenient for confinement of energy in the reactor. Understanding the formation of this layer of rotating plasma is therefore essential.

The High confinement regime depends on the formation of a layer of plasma that has a high, sheared poloidal rotation. This coresponds to a ring of vorticity. We have previously identified such states resulting from the numerical solution of a differential equation [1]

$$\Delta \psi + \frac{1}{2} \sinh \psi (\cosh \psi - 1) = 0 \qquad (1)$$

where ψ is the electrostatic potential divided by the toroidal magnetic field. We have derived this equation from a field theoretical model of two-dimensional plasma and have obtained several convincing confirmation of its validity [2],[3].

The role of the effective Larmor radius, which combines the diamagnetic and the rotation velocities, was investigated using computer solutions of the differential equation describing stationary states. The Ertel's theorem was used to find to what extent the particle density is driven by the vorticity to develop a local maximum in the rotation layer. This corresponds to experimental results on the tokamak DIII-D and in general with the experimental observation that the layer of rotating plasma coincide with a pedestal where the gradient of the density is very high.

The dynamics of the vorticity and density fields was studied and the natural vorticity profiles were determined using Eq. (1). It appears as an effective Larmor radius that combines the diamagnetic velocity and the poloidal rotation velocity.

The field-theoretical formalism we have developed for plasma immersed in strong magnetic field (like in tokamak) has revealed an unexpected aspect: the reduced (Abelian) symmetry of the theory leads to a differential equation that is different of (1) and has the form

$$\Delta \psi = \exp(\psi)(\exp(\psi) - 1)$$

where ψ is the streamfunction. Solving this equation we have found solutions of ring-type for the vorticity, therefore close to the H-mode distribution. It is necessary to investigate the space scales that are typical for such rings. The first estimations show that the sonic Larmor radius is a possible scale, and this means that the Kelvin-Helmholtz event evolving to a rolling spiral can stabilize itself at the solution of the ring-type, since this one is topological, which means protected by a threshold (a lower bound for energy). The subject is important for the understanding of the saturation of the rotation both for H-mode and for the Internal Transport Barrier: the generation of vortices by nucleation in a sheared rotation layer.

Filamentation events in the high confinement regimes

All instabilities of drift-wave type are suppressed since thier growth rate is lower than the shear of the velocity. It is however supposed that in the layer there is a „peeling-balloonning" mode

that leads to the formation of the periodic structures which are similar to the Edge Localized Modes, observed experimentally. We propose a different concept, in which the current-density and the gradient of pressure are not necessarily the main factors. The breaking-up of this layer was investigated using the model of "drop-on-ceil" instability, with the purpose of comparing with the blobs of density that have been observed in several tokamak devices.
We have investigated the stability of the layer of sheared rotation. It is shown that beyond a certain limit of shear, the layer becomes unstable to generation of vortices inside it. The nucleation of vortices is simply the redistribution of the vorticity in the volume, in a way which is more convenient energetically.
However, more interesting from the point of view of the comparison with the experiments is the evolution of the nucleated vortices towards filaments. The filamentation covers the three most important aspects: vorticity, particle density, current density.

The tearing of the density distribution in a layer of current

The current sheets are unstable to the tearing instability and they can be torn apart into strip of current. The geometry adopted by Trubnikov is adequate for studying the tearing of the density distribution in the layer. The width is initially L_0 and it evolves to a profile L which is variable along the direction x of the layer. The coordinate y is perpendicular on the layer in the equilibrium position. **NOTE**. This means that y is *radial* and x is *poloidal*.
The magnetic field has a shear $B = B_x(y) = -B_0 \tanh\left(\frac{y}{L}\right)$. The current density is $j_z = en(v_{iz} - v_{ez})$. Then

$$v_{iz} - v_{ez} = \frac{cB_0}{2\pi enL(t,x)} = \frac{\text{const}}{nL}$$

Introduce a normalized density of plasma $\rho(t,x) = \frac{nL(t,x)}{n_0 L_0}$ and we have the usual density conservation

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v) = 0$$

Under the assumption $v_{e,th} \ll v_{ez}$ we have the equation of motion

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = \frac{1}{nm_i c}\left(-j_z B_y\right) = \frac{e}{m_i c}\left(v_{iz} - v_{ez}\right)\frac{\partial A}{\partial x}$$

We consider that the system is invariant along the z direction which means that the generalized momenta of the electrons and of ions are conserved

$$m_i v_{iz} + \frac{e}{c}A = \text{const}\,,\, m_e v_{ez} - \frac{e}{c}A = \text{const}'$$

The equation of motion becomes

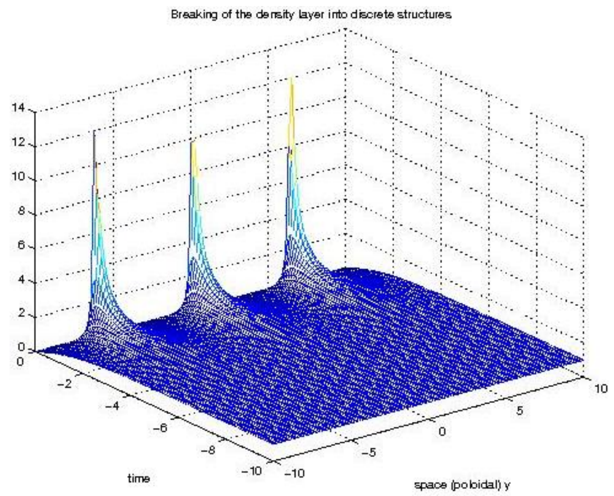
$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = \frac{e}{m_i c}(v_{iz} - v_{ez})\frac{\partial A}{\partial x} = c_0^2 \frac{1}{\rho^3}\frac{\partial \rho}{\partial x}$$

The constant is $c_0^2 = \frac{c m_e}{(m_e + m_i)}\left(\frac{c B_0}{2\pi n_0 L_0}\right)^2$. We note the condition $c_0 \ll v_A$. The two equations

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v) = 0$$

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = c_0^2 \frac{1}{\rho^3}\frac{\partial \rho}{\partial x}$$

are solved using a *hodograph* transformation. The picture below shows the filamentation of a layer of density as results from the solution derived by Trubnikov.



Tearing of the current density layer

It is a study of the *nonlinear* stage of the tearing instability which is made according to the method developed by **Bulanov Sasorov**. The initial state: a plane plasma sheet lying in the $y = 0$ plane with a sheared poloidal (on x) magnetic field. The current flows in the sheet along the z direction. It is assumed that the most important variation of all the quantities in the equilibrium, *unperturbed* state, takes place in the y direction, *i.e.* transversal to the sheet. For slow motion like in the tearing mode, the plasma is assumed neutral and $v_{ex} = v_{ix} = v$ the velocity along the layer, poloidal. The z current is dominated by the electrons, v_{ez} , and this is connected with the initial state by the conservation equation

$$v_{ez} - \frac{e}{m_e c}A = v_{ez}^{(0)}$$

This is actually the conservation of the *generalized electron momentum* along the symmetry direction, z . The equations are

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = -\frac{e}{m_i c}\left(V_z^{(0)} + \frac{e}{m_e c}A\right)\frac{\partial A}{\partial x}$$
$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x}(nv) = 0$$
$$\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} = \frac{4\pi eL}{c}\delta(y)n\left(V_z^{(0)} + \frac{e}{m_e c}A\right)$$

Where $V_z^{(0)} \equiv v_{ez}^{(0)} - v_{iz}^{(0)}$. We need boundary conditions for the z component of the magnetic potential, A. These consists of assuming that at large distances in the transversal direction to the layer, on y, the poloidally oriented magnetic field becomes constant (this is like the Harris profile) $B_0 = \frac{2\pi}{c}en_0LV_z^{(0)}$. Then $A(x,y)\big|_{y\to\pm\infty} = B_0|y| + O\left(\frac{1}{y}\right)$. Integrating the last (for A) equation over the transversal direction, close to the layer, $y \approx 0$, we obtain

$$\left.\frac{\partial A}{\partial y}\right|_{y=0} = \frac{\pi eL}{e}n\left(V_z^{(0)} + \frac{e}{m_e c}A|_{y=0}\right)$$

The following new variables are introduced

$$D \equiv \frac{e}{V_z^{(0)}\sqrt{m_e m_i}}(A - B_0|y|)$$
$$N = \frac{n}{n_0}$$
$$W = \frac{v}{V_z^{(0)}}$$

and a redefinition of the variables $\frac{x}{l} \to x$, $\frac{y}{l} \to y$, $\frac{V_z^{(0)}t}{l} \to t$. Here l is the inhomogeneity scale in the initial perturbation.

The equations become

$$\frac{\partial W}{\partial t} + W\frac{\partial W}{\partial x} = -\frac{1}{2}\frac{\partial}{\partial x}(1+D)^2$$
$$\frac{\partial N}{\partial t} + \frac{\partial (NW)}{\partial x} = 0$$
$$\frac{\partial^2 D}{\partial x^2} + \frac{\partial^2 D}{\partial y^2} = 0 \text{ for } y > 0$$

with the boundary conditions

$$D|_{y\to\infty} = O\left(\frac{1}{y}\right)$$
$$\varepsilon\frac{\partial D}{\partial y} = N\left(1 + D|_{y=0}\right) - 1$$

The solution is determined as a series in powers of the small parameter ε

$$f(t,x,y,\varepsilon) = f^{(0)}(t,x,y) + \varepsilon f^{(1)}(t,x,y) + \varepsilon^2 f^{(2)}(t,x,y) + \ldots$$

In the zeroth order it is derived the relationship between the potential $D^{(0)}$ and the density $N^{(0)}$ in the sheet

$$D^{(0)}(x, y = 0) = -1 + \frac{1}{N^{(0)}(x)}$$

Using this zeroth order relationship we return to the equations: of motion (for W) and of density conservation (for N),

$$\frac{\partial W}{\partial t} + W\frac{\partial W}{\partial x} = -\frac{1}{N^{(0)}}\frac{\partial}{\partial x}\frac{1}{N^{(0)}}$$

$$\frac{\partial N^{(0)}}{\partial t} + \frac{\partial}{\partial x}\left[N^{(0)}W\right] = 0$$

These are drop-on-ceil instability of a system similar to a Chaplygin gas with negative polytropic. A change of variables from the Eulerian variables (t, x) to the Lagrangian variables (t, ξ) ,

$$x = \xi + \Theta(\xi, t)\ , \ W = \left.\frac{\partial \Theta}{\partial t}\right|_{\xi=\xi(t,x)}$$

The plasma density is

$$N = \frac{1}{1 + \partial\Theta/\partial\xi}$$

It is assumed that at the initial time $t = 0$,

$$\Theta(t = 0, \xi) = 0\ ,\ N(t = 0, \xi) = 1$$

The change of variables from Eulerian to Lagrangian variables leads to

$$\left.\frac{\partial W}{\partial t} + W\frac{\partial W}{\partial x}\right|_{Euler} = \left.\frac{\partial W}{\partial t}\right|_{Lagrange} = \frac{\partial^2\Theta}{\partial t^2}$$

and

$$\left.\frac{1}{N}\frac{\partial}{\partial x}\frac{1}{N}\right|_{Euler} = \left.\frac{\partial}{\partial \xi}\left(1 + \frac{\partial\Theta}{\partial\xi}\right)\right|_{Lagrange} = \frac{\partial^2\Theta}{\partial\xi^2}$$

This transforms the equation of motion in Eulerian variables into the equation

$$\frac{\partial^2\Theta}{\partial t^2} + \frac{\partial^2\Theta}{\partial x^2} = 0$$

The complex variable is introduced $s = \xi + it$ and it is seen that the variables

$$\frac{\partial\Theta}{\partial t} = W\ ,\ \frac{\partial\Theta}{\partial\xi} = \frac{1}{N} - 1$$

are harmonic conjugate, which means that also W and $\frac{1}{N}$ are *harmonic conjugate*.

The solution to the differential equation for the function Θ in the complex plane is

$$\Theta(\xi,t) = \mathrm{Re}\left[\int_0^{\xi+it} w(s)ds\right]$$

where the function $w(s)$ is *analytic* and is governed by the condition at the boundary on the *real axis*.

$$w(\xi+it)|_{t=0} = -\frac{\overline{n}(\xi)}{1+\overline{n}(\xi)} + i\overline{v}(\xi)$$

Using the connection between Θ and N and W, and the solution for the harmonic function Θ, one finds

$$N(\xi,t) = \frac{1}{1+\mathrm{Re}[w(s)]} \quad , W(\xi,t) = \mathrm{Im}[w(s)]$$

The solution describes the evolution for $t > 0$ up to the moment when

$$1 + \mathrm{Re}[w(s)] = \begin{cases} 0 & \text{or} \\ \infty & \end{cases}$$

The meaning is: when $\mathrm{Re}[w(s)] = -1$ one has the case that the plasma density becomes *infinite* since the trajectory intersects itself. The other singular situation is $\mathrm{Re}[w(s)] = \infty$ which means that the density *vanishes*, and the plasma sheet is torn. It is mentioned by Bulanov and Sasorov that the situation of one or the other of singularity arises in finite time t.
The solution shows: (1) the sheet is torn apart after a finite time interval and (2) the density vanishes over a finite interval on x.

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