THE ANOMALOUS TRANSPORT IN PLASMAS

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1. Overview

The main research topics for this year have been organized in nine milestones with theoretical and experimental relevance:

1. Influence of the collisions in the gyrokinetic simulations on the particle transport
2. Characterization of the intermittent events and particle transport on the plasma boundary
3. Characterization of long-range correlations and multi-scale physics in L-mode plasmas and during edge improved confinement regimes
4. Analytical expression for power density deposition profile of ICRH
5. Study of variation of the density profile of impurities for different species
6. Test-particle simulations of ion drift in stochastic magnetic fields
7. Test-particles transport in two-dimensional turbulent plasma
8. The construction of a fractional diffusion equation in velocity and position space
9. The study of the saw-tooth crash phenomena using mapping models
10. The study of the test particles' dynamics using mapping models

The first topic study the particle transport by using the statistical properties of the random electric field extracted from numerical simulations. Models that explains the regularities of the statistical properties o the random fields, like self-similarity, were elaborated. These models were used to obtain numerical algorithm for the study of particle transport. Self-similar models modelled the effect of the collisions on impurity transport.

The second topic develops new class of stochastic model was developed, based on our previous result concerning on dimensional reduced model of the edge plasma turbulence. The new class of model is related to the complex version of the previous linear model by a nonlinear mapping, which allows the use of the previous results on the heavy tail exponent of the large particle flux probability with the intermittency effects. The restrictions of the previous linear model were also weakened: the driving noise is very general, not necessary Gaussian and the proof of the main results uses completely new techniques. The dimensionless parameter, the exponent of the probability density function, that characterizes the edge plasma intermittency, was calculated by using functional analytic methods, which leads finally to a simple formula. The phase space of the new model is extended, thus allowing providing a more general reduced stochastic model of the relaxation oscillations. The results are useful in the evaluation of the effects of intermittent heat loads to the plasma facing components.

In topic 3 the long-range correlations were studied by extension of the previous results to the study of random multiplicative non-linear processes in the complex plane. Particle transport in turbulent electrostatic field with long-range correlations was studied by the spectral methods from the abstract ergodic theory. A natural explanation of the emergence of the long range spatial correlation by discretized version of stochastic linear partial differential equation was elaborated.

In topic 4 is proposed an analytic expression for a well description of the power deposition profile in the case of Ion Cyclotron Resonance Heating of plasma in tokamak fusion device.

The fifth topic studies the dispersion equation for ITG instability in plasma with ICRH. In the case of plasma with multispecies ions was analysed the effects of harmonics cyclotron resonance waves for one ion species on the other ion-species.

In topics 6 and 7 the test particle diffusion in an electromagnetic stochastic field was studied in the framework of the decorrelation trajectory method (DCT). In this kind of stochastic field we have started the study of the test-particles in an electromagnetic stochastic field, in order to determine the mean square displacements and the radial and poloidal diffusivities using the numerical method and the decorrelation trajectory method. The comparison of the results obtained by these two methods might then allow us to confirm the DCT approach results. These results are important for "Validation of physics-based transport models" and "Plasma edge characterization and modelling", objectives for ITER.

In topics 8, 9 and 10 some features of the anomalous transport were studied using mathematical models which involve the fractional calculus and the hysteresis technique. The aim of the research is to understand if the memory effects might be responsible for some phenomena observed in experiments, for example for the saw tooth crash of the central temperature in ASDEX-Upgrade tokamak.

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[2] G. Steinbrecher, X. Garbet, B. Weyssow, "*Weak Convergence to Stationary Distributions in Heavy Tail Models*", sent to Stochastic Models, partial results in Annals of the Univ. Craiova "Physics AUC", **19**, 107-123 (2009).

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[14] M. Negrea, I. Petrisor, B. Weyssow and H. Isliker, *Aspects related to the magnetic field lines diffusion in tokamak plasma*, invited lecture at 8th School of Fusion Physics & Technology, 6 – 11 April 2009, Volos, Greece.

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[16] H. Isliker, A. Vogiannou, L. Vlahos, M. Negrea, I. Petrisor and B. Weyssow, *Test-particle simulations of ion drift in stochastic magnetic fields*, poster session at 36th EPS Conference on Plasma Physics, June 29 - July 3, 2009, Sofia, Bulgaria.

[17] M. Negrea, I. Petrisor, D. Constantinescu, H. Isliker, A. Vogiannou and L. Vlahos, *Aspects of the Diffusion of Ions in Tokamak Plasma*, poster session at 13th European Fusion Theory Conference, October 12 -15, 2009, Riga (Latvia).

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2. Detailed results

2.1. Influence of the collisions in the gyrokinetic simulations on the particle transport

This objective is continued from 2008. In the previous stage, the statistical properties of the stochastic electrostatic field were studied. In this period it was found that it is possible to approximate the stochastic electrostatic field by a 3 dimensional generalization of the classical one-dimensional self–similar processes with stationary and independent increments. The mathematical foundations of the numerical approximation of the particle transport in electrostatic field having the properties of generalized self-similar processes were studied. New models for the explanation of the generation of the random field with the characteristics of this new class of self-similar random field were elaborated. These models predict that by a modification of the collision terms, the statistical properties o the random field is only slightly changed. In the first class of models the random field is locally generated by a special case of the stochastic processes exposed in ref. [1]. In the second class by the use of the generalized Central Limit Theorem the stable random field is generated by a superposition of more general class of processes, studied in refs. [2-4]. Further analysis of the time evolution of the instabilities and their localised structure will be performed in order to discriminate among the previous coarse- grained models.

The elaboration of the mathematical methods for the interpretation of the first principle gyrokinetic simulations is important for the optimisation of the strategy of the time consuming first principle simulations, necessary for the optimisation of the large tokamaks, like ITER and DEMO.

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2.2. Characterization of the intermittent events and particle transport on the plasma boundary

The study of the intermittent events in the plasma boundary was studied by extending previous results from refs. [1, 2]. The new feature of this model is the very general character of the driving multiplicative noise that is not necessary a Gaussian. The driving noises could be generated by any abstract ergodic dynamical system, which has at least the statistical mixing properties of a K system. In contradistinction to the previous random linear models, this model allows a weakly non-linear additive term. The intermittency of the edge plasma was characterized numerically by the singularity structure of the stationary probability density function of the particle flux fluctuations. In this class of models from refs. [3, 4, 5], the singularity exponent of the stationary probability density function at zero value, which corresponds to the laminar periods of the intermittent behavior, quantifies the intermittency effects. Remarkable universality of this exponent was obtained: it is not sensitive on the details of non-linear additive term. It is completely determined by the statistical properties of the dominating linear term, in particular by the mean value of its real part and zero frequency component of the power spectrum of its fluctuations. The corresponding stochastic differential equation with driving complex valued stochastic processes $\varsigma(t)$, $\phi(t)$, for the complex valued function Z(t), is $dZ(t)/dt = (a+\varsigma(t))Z(t) - Z(t)|Z(t)|^2 \phi(t)$. Here a is a complex constant, whose real part is related to the stability threshold of the edge plasma instabilities.

Define, according to ref.[5] the function $Y(T) = \int_0^T \mathrm{Re}(\varsigma(t))dt$ and $D = \lim_{T\to\infty} \left\langle [Y(T)]^2 \right\rangle / T$.

The main result is the analytic formula for the numerical value of the exponent of the singularity λ is given by $\lambda = \mathrm{Re}(a)/D$.

The study of the statistical properties of the intermittent events in the edge plasma turbulence is essential in the evaluation of the effects of the intermittent heat loads on the plasma facing components of large tokamaks.

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2.3. Model for edge plasma turbulence.

Characterization of long-range correlations and multi-scale physics in L-mode plasmas and during edge improved confinement regimes.

The effect of the long-range spatial correlations of the stochastic electric field on the impurity transport. Particle transport in the frozen turbulence with long-range correlations.

The problem of transversal particle transport in constant magnetic field and random electric field was studied, with special focus on the evaluation of the fraction (denoted by T) of permanently trapped particles and its dependence on the power law exponent α of the correlation decay. The correlation function of the electric field in this model was supposed isotropic, homogenous and with long-range spatial correlations: power law decay at infinity with exponent less or equal 2. The frozen turbulence approximation was used, in order to approximate the long range time correlations. By suitable Hilbert formalism rigorous bounds on the fraction of trapped particles was obtained [6]. In this way the extensive Monte-Carlo type simulations, that are necessary to solve numerically this class of problems is replaced by a deterministic problem: numerical minimization of a class of analytic functions of many variables. A specific version of the Fletcher-Reeves method was adapted and a C++ code was tested.

Stochastic models of the long-range correlations, self-similarity, multi-scale physics, intermittency, in the plasma edge plasma turbulence.

Previous stochastic linear low-dimensional models of the edge plasma turbulence from refs. [1-4] was used for the elaboration of a new mechanism that contributes to the L-H transitions. This new class of models based purely on stochasticity has a minimal set of assumptions, and give unified mathematical explanation for the occurrence of the intermittent behaviour together to the transition to the H mode.

The main shortcoming of this class of the models is the reduced number of components. Consequently we begin the study of a new class of stochastic models [7-8]: Discrete Random Linear Partial Differential Equations (DRLPDE).

These models appear in the stochastic version of the linear instability studies. This category of DRLPDE also describes the effect of the small spatial and temporal scale turbulence to the large-scale evolution of the physical system. In order to be able to study higher dimensional stochastic models by analytic methods, we used in [6-7] the powerful new results from the theory of Anderson localization [9]. This higher dimensional DRLPDE are interesting also because they relate the local, disordered random fluctuations to the global statistical characteristics of the physical system, they models the multi scale physics. We studied a simplified model: classical Hamiltonian linear hyperbolic equations with random potential term ("Klein-Gordon Equation with random "mass" term"). The main motivation was that exactly the Hamiltonian systems in higher dimensions are most exposed to destabilization by random perturbation. The main results are:

1. For a stochastic system with N components, it is possible to obtain a closed set of soluble linear deterministic differential equations for the moments of order p.

2. The k=0 mode is always destabilized.

3. In dimension 1 and 2 all of the modes are destabilized, but the modes with high k has a smaller positive Liapunov exponent

4. In 3 dimensions, for low noise intensity, the modes with sufficient high k remain stable. This is a mechanism for the occurrence of the long-range spatial correlations, generated by a completely disordered noise. It is an interesting multi-scale physics effect. This is also an interesting linear stochastic model that gives a qualitative description of the inverse cascade.

This class of models of the generation of the long–range spatial correlations, spatial structures from the parametric amplification of a completely disordered random field characterized by temporal and spatial white noise, give a new, analytic method for the explanation of the main qualitative features of the anomalous particle transport in large tokamaks.

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2.4. Analytical expression for power density deposition profile of ICRH

Resonant interaction of ions with ICRF waves leads both to heating the plasma and spatial transport of the resonating ions. The current and torque provided by ICRH depend on the concentrations of heated species and also on the positions of the resonance layers. Recently works analyses the resonant ion cyclotron interactions in tokamaks and influence of the radio frequency ponderomotive force on anomalous impurity transport in tokamaks [1, 2]. In both issues the resonance layer position and the profile of power absorption plays an important role for stability and confinement properties of the plasma.

In the following was analyzed the profile of power absorption by tritium in a deuterium-tritium plasma heated by ICRF. In many works this profile is considered for simplicity as a Gaussian, but experiment measurements and numerical simulation reveal a more complex profile [3].

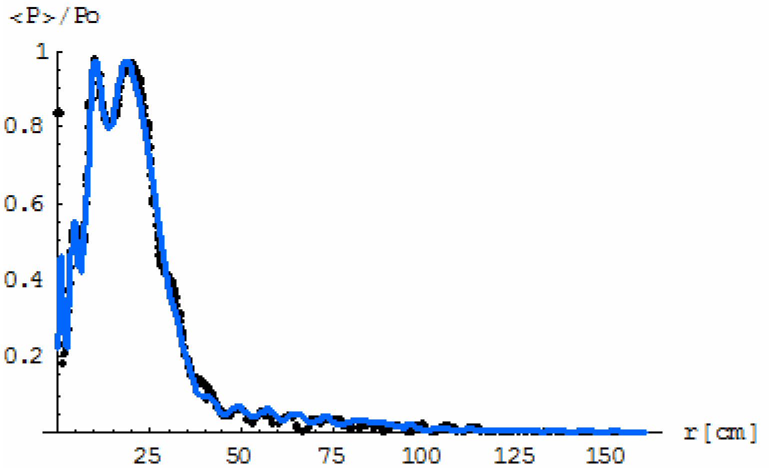

Figure 1. Profile for normalized power density absorption obtained with new proposed expression (continuous line) and the points obtained from numerical simulation.

This profile can be well described by the following proposed expression [4],

$$< P > / P_0 = 0.8 Exp\left[-\frac{(r-0.03R_0)^2}{5.6(\Delta X)^2} \right] + 0.21 Exp\left[-\frac{3(r-0.017R_0)^2}{(\Delta X)^2} \right]$$
$$+0.3Exp[-7r/a]\left(\sin\left[\exp\left[2.7/(1+r/a)^6\right]\right]\right)^2 + 80(r/a)^2 \exp[-7r/a]\left[J_1(80r/a+3/2)\right]^2$$

where r is the toroidal radial coordinate, R_0 is the major radius and $\Delta X = R_0 \frac{k_{\|} v_{thi}}{n\Omega_{ci,0}}$ is the width determined by Doppler shift of thermal particles; the plot is given in figure 1.

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2.5. Study of variation of the density profile of impurities for different species

In the present study were assumed the following conditions:

- Radio-frequency waves are in Ion Cyclotron Range. We are interested in effects of both heating and ponderomotive force;
- The equilibrium magnetic field is described by the axisymmetric model for a large aspect ratio tokamak with circular concentric magnetic surfaces
- The ITER-like plasma is multi-species plasma. The species are electrons, Deuterium, Tritium, alpha particles and one other impurity species (like Argon).
- We are interested in the impact of Ion Temperature Gradient (ITG) modes on ion density profile in tokamak.

The influence of radio frequency ponderomotive force on anomalous impurity transport in tokamak was studied recently – see ref. [1] in fluid formalism with trace impurity approximation where the normalized impurity density peaking factor is almost constant relative to ponderomotive force term. The result of the balance between convective and diffusive processes disagrees with experimental observation but agree with other fluid or kinetic studies. Consequently we decide to study the influence of radio frequency waves on anomalous impurity transport in tokamak for multi-species plasma using the kinetic formalism. The ensemble averaged distribution function is assumed as the summation between a Maxwellian part and the deviation from Maxwellian where the last depends on both turbulence and particles - RF wave interaction.

The fluctuating part of the distribution function is linearly approximated as a summation between the solution of kinetic equation for turbulence in absence of RF waves and the solution of the kinetic equation in presence of RF-waves but in absence of turbulence.

We used for the ITG-instability a description similar with references [2, 3] but adding more ion species and also the interaction with RF waves. For electrons was assumed an adiabatic response and for ion species both adiabatic and non-adiabatic response was evaluated using the ballooning transform of the perturbed distribution function. The ion density perturbation profile due to ITG in plasma with ICRH was obtained [4, 5].

In our model we have obtained from the electro-neutrality condition the dispersion equation for the frequency. This equation which contains the dependence of the electrical charge of ion species, the characteristics of ITG instability and those of the RF wave externally launched was studied and compared with similar equations obtained in fluid description. The significant importance of the resonant interaction in the regions where the gradient of the density profile of an ion species is important was revealed. This region is near to the edge for some species but for other species this can be outside of the plasma. We emphasize that for this later species the impact of coupled action RF wave – ITG instability can produce a significant modification in their densities. The parameters specific to ITER has been considered in numerical evaluations and the conclusions refer to the installations like ITER. In conclusion, when one ion species is heating by corresponding resonant cyclotron waves, the later interact also with other ion species and can modify their density profile.

The present work was partially done at Chalmers University of Technology, Gothenburg, Sweden and will continue in 2010.

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2.6. Test-particle simulations of ion drift in stochastic magnetic fields

We have studied the influence of stochastic magnetic fields on ion diffusion, using a drift approximation in slab geometry, and applying a stationary stochastic magnetic field on top of a uniform background field. The stochastic magnetic field follows a Gaussian in its distribution function and obeys a prescribed auto-correlation function with given correlation length. The running diffusion coefficients of the ions are determined with the use of test particle simulations in the three dimensional environment, for different levels of turbulence (varying the drift Kubo number and with a fixed magnetic Kubo number). The results of the test-particle simulations were compared to the results obtained for the same physical system by the semi-analytical decorrelation trajectory method (DCT). By construction, the magnetic field is periodic in all three directions, and particles leaving the simulation box are re-injected at the plane opposite to the one through which they leave.

The stochastic magnetic field (figure 2) is generated on a grid with 643 grid-points, with a grid-spacing such that the grid-size equals 9 correlation lengths in each direction. In conclusion we find that diffusion is of normal nature (see e.g. the radial running diffusion coefficient in figure 3 and a typical trajectory in figure 4). The diffusive process slows down with both, increasing strength of the magnetic perturbation, and increasing mass of the ions considered, respectively.

The values of the diffusion coefficients practically coincide with the results yielded by the DCT we find though a different scaling of the diffusion coefficient with the drift Kubo number [1-7]. Work was done by M. Negrea and I. Petrisor in collaboration with Dr. B. Weyssow from

Universite Libre de Bruxelles and H. Isliker, A. Vogiannou and L. Vlahos from University of Thessaloniki, Greece.

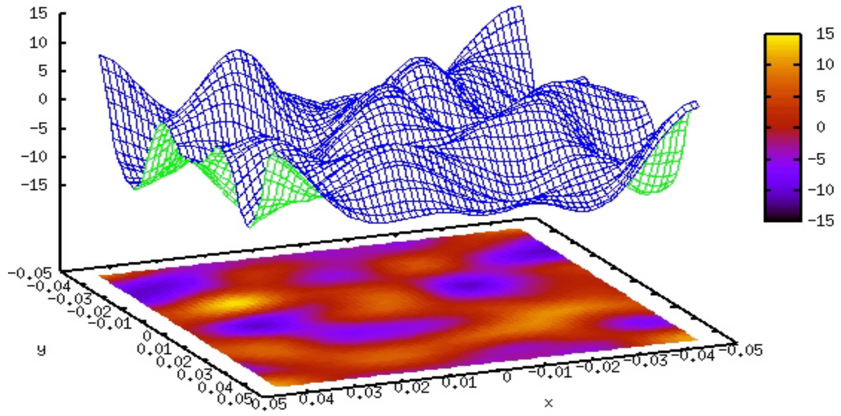

Figure 2: The stochastic magnetic field

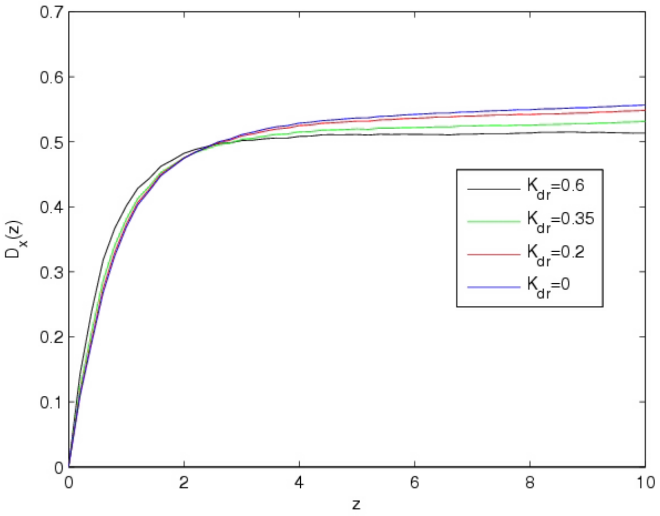

Figure 3: Radial diffusion coefficient for different values of the drift Kubo number and for the magnetic Kubo number equal to unity

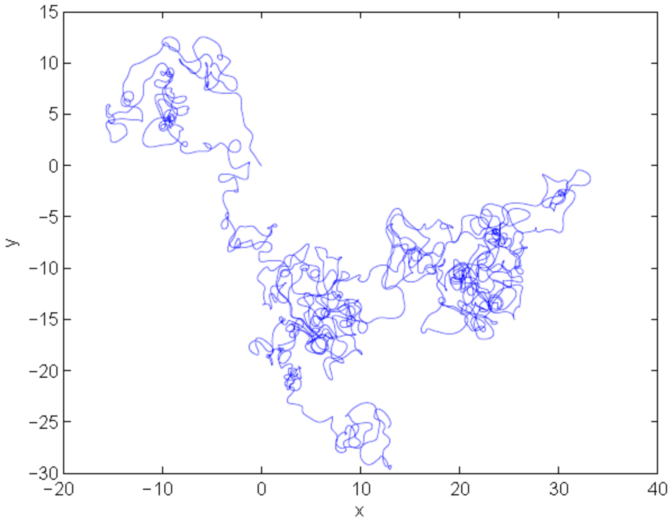

Figure 4: Typical trajectory for the test particle

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2.7. Test-particles transport in two-dimensional turbulent plasma

The transport of charged test particles in two-dimensional turbulent plasma is studied using numerical simulations. The plasma is considered in the magneto-hydrodynamic approximation and the turbulence level is fixed using an external mechanical force. The self-consistent electromagnetic field generated by the plasma acts on the test particle via the Lorentz force while the interaction between the test particles and the plasma constituents is modelled as a drag force. In the absence of collisions, the mean electric field accelerates the particles. This gives a super-diffusive behaviour in the velocity space and a super-ballistic regime for the position.

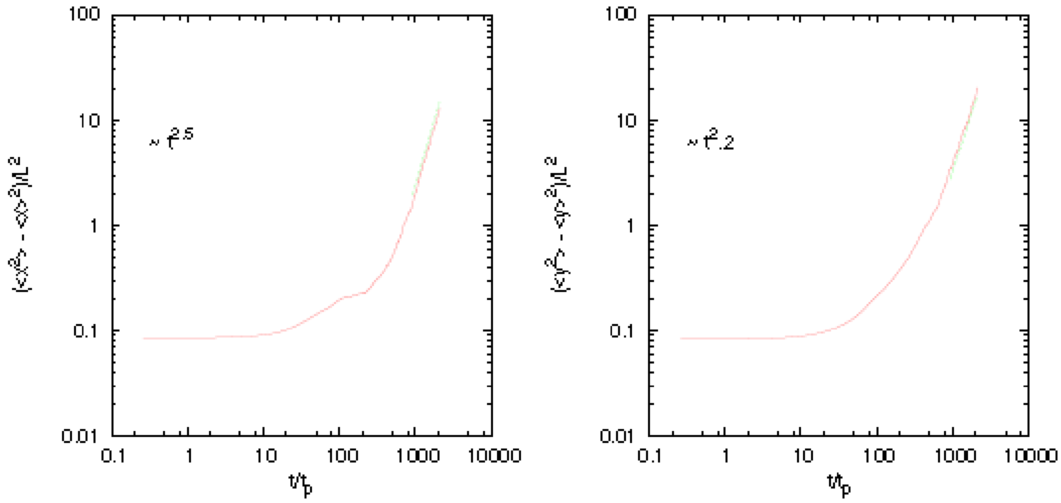

Figure 4: Mean square displacements in radial (left) and poloidal (right) directions.

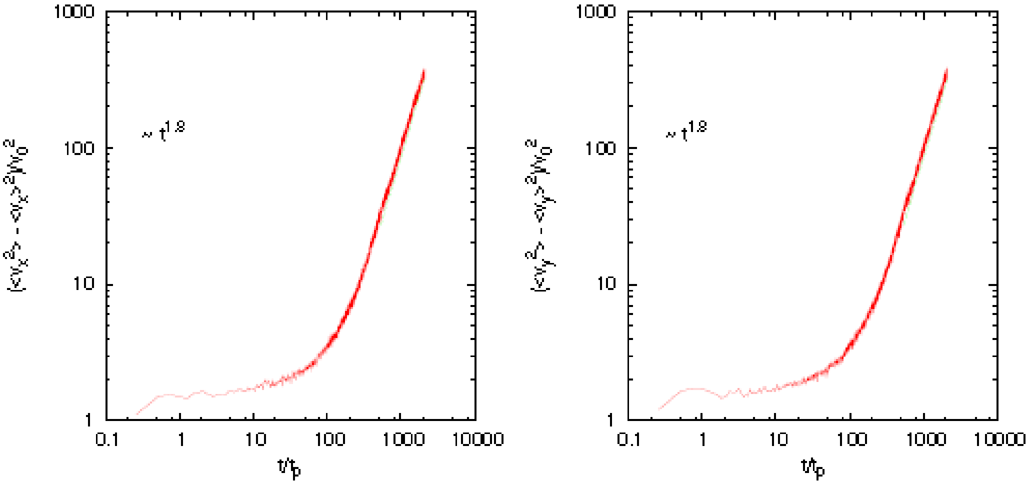

Figure 5: Velocity mean square displacements in radial (left) and poloidal (right) directions.

A large value for the coupling between the electromagnetic field and the particle gives a high trapping rate due to the magnetic field. Particles will tend to follow the magnetic field lines and the transport regime is given by the turbulent level of the magnetic field lines. In figures 4 and 5 we have represented the spatial mean square displacement and velocity mean square displacement, respectively. The electric field accelerates the particles, increasing their kinetic energy and lowering the coupling (particle velocity will increase compared to the field reference Alfvén velocity). The presence of collision should decrease the level of energy gain in the unit of time. The work is in progress and was done by Negrea Marian and Petrisor Iulian in collaboration with Dr. D. Carati, B. Teaca and C. Lalescu from Universite Libre de Bruxelles.

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2.8. The construction of a fractional diffusion equation in velocity and position space

We started our study from the already developed Continuous Time Random Walk equations. In a continuous time random walk model, combined in position and velocity space, we determined the fluid limit of the equations and we derived and analyzed a fractional diffusion equation for the model. We will use this equation for the calculation of the particle and heat diffusivities or pinch velocities specific to the characterization of the transport processes. As a second activity, the fractional diffusion equation in position and velocity space will be applied to the electromagnetic environment described in the milestone described above “Numerical simulation of test particle transport and comparison with theoretical models”, in order to model the results of the test particle simulations. The fractional diffusion equation was used in order to describe a nonlocal type of transport: the fluid limit of “microscopic” kinetic transport processes without characteristic spatial and temporal scales [1, 2]. These transport processes, that are known as Continuous Time Random Walk, generalize the standard Brownian motion by allowing trapping events and large displacements known as Levy flights. The fractional diffusion models depend on the usual parameters α and β determine the order of the fractional derivatives in space and time, respectively. In our study these parameters were varied in order to take into account the non-Markovian memory effects (when $\beta \neq 1$) and spatial anomalous effect (when $\alpha \notin \{1,2\}$). This objective will be continued in 2010 in collaboration with H. Isliker, A. Vogiannou and L. Vlahos from University of Thessaloniki, Greece.

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2.9. The study of the saw tooth crash phenomena using mapping models

The aim of our studies is to point out some mechanisms that are susceptible to govern the saw tooth oscillations in tokamak. An approach based on the Hamiltonian description and the mapping technique was used in order to reconstruct the evolution of the magnetic field during the saw tooth instability. Two magnetic configurations, specific to Tore-Supra and to ASDEX-Upgrade tokamak were studied. In the first model [1], a perturbation that contains not only the m/n=1/1 mode, but also higher harmonics, was considered and the role of these additional perturbations during the partial saw tooth crash was observed [2].

For the second model [3], which represents an accurate description of the magnetic field configuration during the saw-tooth instability in ASDEX-Upgrade tokamak, the perturbed safety factor was reconstructed for various values of q_0 and various amplitudes of the perturbations. The influence of the parameters of the systems (the minimum value of the safety factor, the flatness coefficient and the amplitude of the perturbations) on the reconnection process was systematically studied. The relationships among our results, current theoretical models and experimental data were also studied.

We proved that the scenario proposed for the saw tooth reconnection in ASDEX-Upgrade tokamak is motivated by general properties of Hamiltonian systems describing magnetic configurations with a monotonous safety factor, so it is reproducible in many situations. It was also pointed out that it is crucial that the modes (1, 1), (2, 2), (3, 3) have the same rotation number (see figures 6 and 7), so they are situated in the same spatial region. It was proved that this scenario cannot be applied in the case of magnetic configurations with a reversed shear safety factor, due to the specific properties of the safety factor and a scenario for the saw tooth magnetic reconnection was proposed in this case [4, 5].

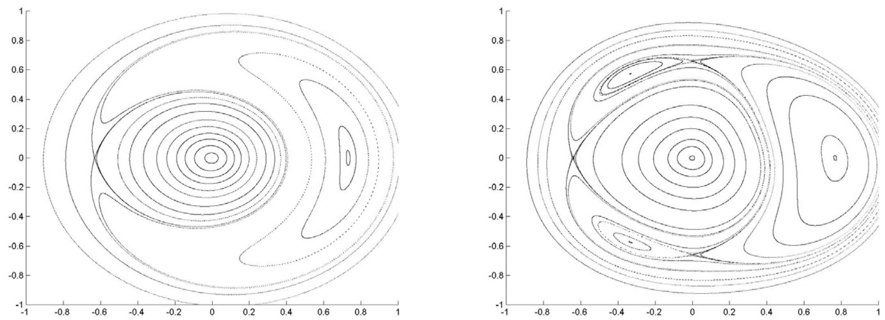

Figure 6: The Poincare section for the monotonous perturbed safety factor is represented for $q_0=0.7$ *and* $\varepsilon=0.5$*. In the left subplot (a) only the (1, 1) mode is excited and in the right subplot (b) are excited the modes (1, 1), (2, 2), (3, 3).*

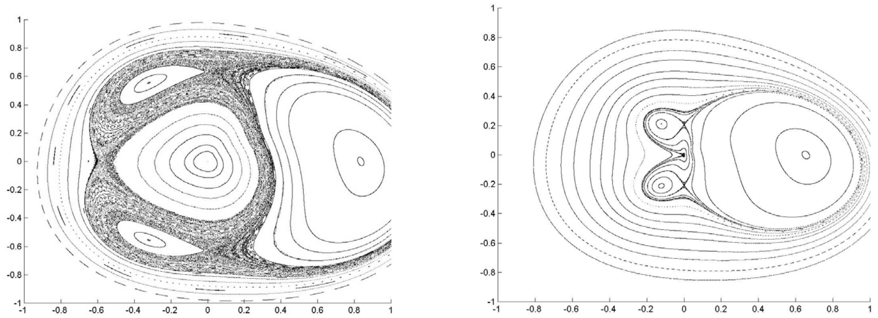

Figure 7: The Poincare section for the monotonous perturbed safety factor is represented for $q_0=0.7$ *[subplot (a)] and* $q_0=0.9$ *[subplot (b)]. For both subplots* $\varepsilon=1$ *and the modes (1,1), (2,2), (3,3) are excited.*

The study of the saw tooth oscillations of the central temperature in ASDEX-Upgrade tokamak was continued. Two differential models were used in order to reproduce some experimental data. In the first model we considered the memory effect of hysteresis type and it was proved that the value of the hysteresis parameter can be chosen such that the model reproduces correctly the two time scales of the saw tooth crash: the slow rise time (~7 ms) and the rapid crash time (~50 μs) [3]. The second model is a system of differential equations derived by K. Lackner and H. Zohm for the minimal saw tooth model. For this model we accomplished a detailed analysis of the bifurcations that occurs for various values of the parameters (which are the conductivity, the permeability of the free space, the diffusivity) and we pointed out the physical relevance of the mathematical results [4].

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2.10. The study of the test particles’ dynamics using mapping models

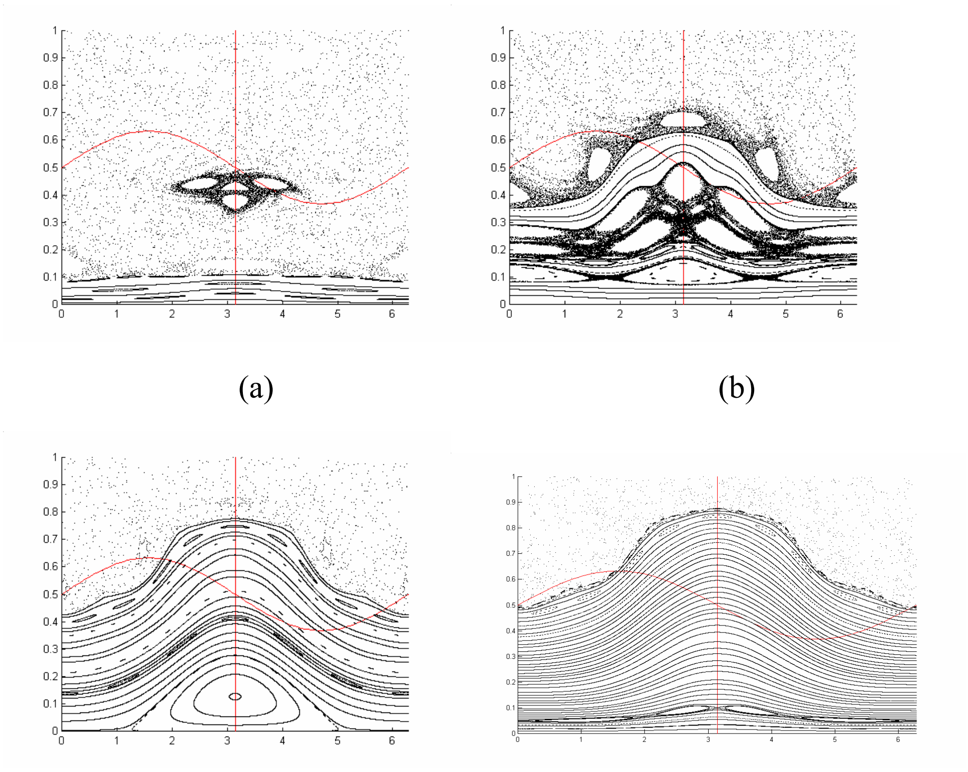
In the previous work the mapping technique was used to describe the magnetic configurations specific to Tore Supra and ASDEX Upgrade.

In the present work it was proved that a magnetic internal transport barrier exists in the low shear region even for monotonous safety factors. This result confirms the experimental observations and the previous intuitive explanations. It was shown that the reversed magnetic shear is not compulsory for the formation of the transport barrier. However, in the reversed shear configuration the transport barrier is more robust, due to the specific topology of the magnetic field involved in the reconnection process. The effect of the amount of flatness of the safety profile in the vicinity of its zero shear point was also studied using the flatness coefficient. The beneficial effect of the vanishing of the shear profile was shown to be increased if the radial extent of the low-shear region is increased due to a flatter q-profile.

The minimum value of the unperturbed safety profile in the low shear region, q_0, also influences the robustness of the transport barriers. The case when it is an integer number is extremely important because it can be correlated with experimental observations.

General results cannot be obtained because the system strongly depends on the perturbations. However, it was proved that, for small and moderate perturbations, a transport barrier is formed when q_0 is a little bit smaller than the main integer values $q_0=1$, $q_0=2$, $q_0=3$, which also confirm the experimental data [1-3].

In order to study the motion of charged particles in the same magnetic configurations, the global drift wave map [4], was adapted. The aim of the work was to study the particles internal transport barriers in the low shear magnetic regions, even in the absence of the reversed shear. Specific safety factors and magnetic perturbations were considered in the general model. Two different measures of a global transport (the running diffusion coefficient and the exit time diffusion coefficient) were computed and compared. The relation between these two measures of transport and the formation of a transport barrier was pointed out: the formation of a transport barrier is associated with a drop of these quantities (see figure 8). The milestone will continue in 2010.



(a) (b)

(c) (d)

Figure 8. The transport barrier (provided that it exists) intersect the regular (red) curve. It is larger when the flatness f coefficient is larger. (a) f=1 (monotonous safety factor): there is not an Internal Transport Barrier (ITB). (b) f=2(reversed shear); (c) f=3 (monotonous safety factor); (d) f=4 (reversed shear): the ITB intersects the regular curve.

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