Trapping in frozen electrostatic turbulence.

György Steinbrecher Assoc. MEdC, University of Craiova Romania

Motivation

- Frozen turbulence is the extreme case of long range temporal correlations.
- The stable dynamical systems are destabilized by very small random perturbations with long range temporal correlations. Steinbrecher, G.; Weyssow, B. Phys. Rev. Lett. 2004, 92, 125003-1 -125003-4.

The physical model

- Constant magnetic field. Frozen electrostatic turbulence, random electric field.
- 1 degree of freedom, autonomous Hamiltonian dynamical system.
- Randomness =>non trivial problem.

The physical model

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- M. Vlad, F. Spineanu, J. H. Misguich, R. Balescu, Physical Review E, 67, 026406 (2003).
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The random electric field

- Long range spatial correlations in random electric field.
- Gaussian with mean zero.
- Self similarity.

The problem

- Consider that the initial position of the particles are distributed randomly, with a given probability density.
- The particles moves according some Hamiltonian equation with random parameters.
- Arnold, L. Random Dynamical Systems; Springer, Berlin, 1998.
- COMPUTE the fraction of trapped particles in a given domain.

The random electric field

- Generalization of the one dimensional fractional Brownian motion.
- Gaussian, homogenous.
- Plasma physical analogue of the Harrison-Zeldovich (scale free) mass distribution from astrophysics.
- RANDOM CHARGE, RANDOM POTENTIAL DIFFERENCE

Fractional Brownian motion, Hurst exponent H

1. The increments $W_{\omega}(t_2) - W_{\omega}(t_1)$ are centered Gaussian processes.

2. Invariance under time translation:

$$W_{\omega}(t_2+a) - W_{\omega}(t_1+a) \stackrel{d}{=} W_{\omega}(t_2) - W_{\omega}(t_1)$$

3. Self similarity $W_{\omega}(\lambda t_2) - W_{\omega}(\lambda t_1) \stackrel{d}{=} \lambda^H [W_{\omega}(t_2) - W_{\omega}(t_1)]$

Fractional Brownian charge, Hurst exp. H

- 1. $W_{\omega}(B)$ is centered Gaussian.
- 2. It is invariant under translations and rotations:
- 3. The scale free property, self similarity.

For any for any collection of sets $B_1, ..., B_n \subset \mathbb{R}^2$, $\{W_{\omega} [B_1], ..., W_{\omega} [B_n] \stackrel{d}{=} \lambda^{-H} \{W_{\omega} [\lambda B_1)], ..., W_{\omega} [\lambda B_n]\}$

Generalization II, 2- dim $\mathbf{x}=(\mathbf{x},\mathbf{y})$

Random Gaussian potential differences , Hurst index ${\cal H}$

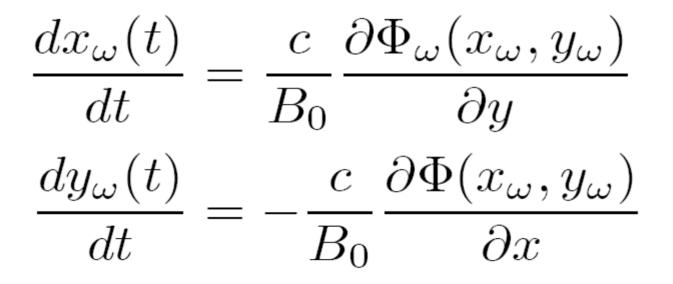
- 1. $\Phi_{\omega}(\mathbf{x}) \Phi_{\omega}(\mathbf{y})$ is centered Gaussian
- 2. Euclidean invariance: g = translation or rotation, then

$$\Phi_{\omega}(g\mathbf{x}) - \Phi_{\omega}(g\mathbf{y}) \stackrel{d}{=} \Phi_{\omega}(\mathbf{x}) - \Phi_{\omega}(\mathbf{y})$$

3. Self-similarity: $\Phi_{\omega}(\lambda \mathbf{x}) - \Phi_{\omega}(\lambda \mathbf{y}) \stackrel{d}{=} \lambda^{H} [\Phi_{\omega}(\mathbf{x}) - \Phi_{\omega}(\mathbf{y})]$

Equation of motion

Random potential $\Phi_{\omega}(x, y)$. Hamiltonian formalism



The trapping fraction T_A

Ergodic theorem

- Uniform distribution of the initial positions.
- Mixed average: over time **t** and the ensemble of the random potential <...>.
- The Hamiltonian form =>von Neumann mean ergodic theorem=>
- Variational principle for computing the trapping fraction.

$$T_{A} = \frac{1}{s} \inf_{g_{\omega}(x)} J\{g_{\omega}(.)\}$$

$$J\{g_{\omega}\} = \int_{\mathbf{R}^{2}} \left\langle \{\chi_{A}(x) - [g_{\omega}, \Phi_{\omega}]\}^{2} \right\rangle_{\omega} d^{2}\mathbf{x}$$

$$g_{\omega}(\mathbf{x}) = arbitrary \ stochastic \ function$$

$$[g_{\omega}(\mathbf{x}), \Phi_{\omega}(\mathbf{x})] \quad Poisson \ bracket$$

Upper bound, trial functions 1

$$g_{\omega}(\mathbf{x}) = \sum_{n=1}^{N} f_n(\mathbf{x}) h_{n,\omega}$$

$$f_n(\mathbf{x}) = arbitrary functions$$

$$h_{n,\omega} = u_n \{ \Phi_{\omega}(\mathbf{a}_1), \Phi_{\omega}(\mathbf{a}_2), ..., \Phi_{\omega}(\mathbf{a}_M) \}$$

$$\mathbf{a}_i = arbitrary points in plane$$

Trial functions 2

$$g_{\omega}(\mathbf{x}) = \sum_{n=1}^{N} [\Phi_{\omega}(\mathbf{x}), f_n(\mathbf{x})] f_n(\mathbf{x}) h_{n,\omega}$$
$$f_n(\mathbf{x}) = arbitrary functions$$

 $h_{n,\omega} = u_n \{ \Phi_{\omega}(\mathbf{a}_1), \Phi_{\omega}(\mathbf{a}_2), ..., \Phi_{\omega}(\mathbf{a}_M) \}$

 $\mathbf{a}_i = arbitrary points in plane$

Numerical method

- Set 2 of trial functions
- The x functions vanishes outside the domain A.
- Optimisation with parameter a in the scaling f(x)->f(ax)
- Domain A with radial symmetry
- In the case of self similar random potential the trapping coefficient is independent of radius.

Result, upper bound

Upper bound on trapping fraction

