Trapping in frozen electrostatic turbulence.

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Motivation

- Frozen turbulence is the extreme case of long range temporal correlations.
- The stable dynamical systems are destabilized by very small random perturbations with long range temporal correlations. Steinbrecher, G.; Weyssow, B. Phys. Rev. Lett. 2004, 92, 125003-1 - 125003-4.

The physical model

- Constant magnetic field. Frozen electrostatic turbulence, random electric field.
- 1 degree of freedom, autonomous Hamiltonian dynamical system.
- Randomness =>non trivial problem.

The physical model

- Balescu, R. Aspects of Anomalous Transport in Plasmas; Institute of Physics Publishing, Bristol, 2005.
- Vlad M., Spineanu F., Misguich J.H., Balescu R., Physical Review. E 67 (2003). 026406.

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The random electric field

- Long range spatial correlations in random electric field.
- Gaussian with mean zero.
- Self similarity.

The problem

- Consider that the initial position of the particles are distributed randomly, with a given probability density.
- The particles moves according some Hamiltonian equation with random parameters.
- Arnold, L. Random Dynamical Systems; Springer, Berlin, 1998.
- COMPUTE the fraction of trapped particles in a given domain.

The random electric field

- Generalization of the one dimensional fractional Brownian motion.
- Gaussian, homogenous.
- Plasma physical analogue of the Harrison-Zeldovich (scale free) mass distribution from astrophysics.
- RANDOM CHARGE, RANDOM POTENTIAL DIFFERENCE

Fractional Brownian motion, Hurst exponent H

1. The increments $W_\omega(t_2) - W_\omega(t_1)$ are centered Gaussian processes.

2. Invariance under time translation:

$$W_\omega(t_2 + a) - W_\omega(t_1 + a) \stackrel{d}{=} W_\omega(t_2) - W_\omega(t_1)$$

3. Self similarity $W_\omega(\lambda t_2) - W_\omega(\lambda t_1) \stackrel{d}{=} \lambda^H [W_\omega(t_2) - W_\omega(t_1)]$

Fractional Brownian charge, Hurst exp. H

1. $W_\omega(B)$ is centered Gaussian.

2. It is invariant under translations and rotations:

3. The scale free property, self similarity.

For any for any collection of sets $B_1, .., B_n \subset \mathbb{R}^2$,

$$\{W_\omega[B_1], ..., W_\omega[B_n]\} \stackrel{d}{=} \lambda^{-H} \{W_\omega[\lambda B_1)], ..., W_\omega[\lambda B_n]\}$$

Generalization II, 2- dim $\mathbf{x}=(x,y)$

Random Gaussian potential differences , Hurst index H

1. $\Phi_\omega(\mathbf{x})-\Phi_\omega(\mathbf{y})$ is centered Gaussian

2. Euclidean invariance: $g = \textit{translation or rotation}$, then

$.\Phi_\omega(g\mathbf{x})-\Phi_\omega(g\mathbf{y}) \stackrel{d}{=} \Phi_\omega(\mathbf{x})-\Phi_\omega(\mathbf{y})$

3. Self-similarity: $\Phi_\omega(\lambda\mathbf{x})-\Phi_\omega(\lambda\mathbf{y}) \stackrel{d}{=} \lambda^H\left[\Phi_\omega(\mathbf{x})-\Phi_\omega(\mathbf{y})\right]$

Equation of motion

Random potential $\Phi_\omega(x, y)$.
Hamiltonian formalism

$$\frac{dx_\omega(t)}{dt} = \frac{c}{B_0}\frac{\partial \Phi_\omega(x_\omega, y_\omega)}{\partial y}$$

$$\frac{dy_\omega(t)}{dt} = -\frac{c}{B_0}\frac{\partial \Phi(x_\omega, y_\omega)}{\partial x}$$

The trapping fraction T_A

$$
\begin{aligned}
T_A &= \left\langle \lim_{t\to\infty} \frac{1}{t} \int_0^t \left[\int_{\mathbf{A}} \chi_A(g_\omega^\tau(\mathbf{x}))d^2\mathbf{x} \right] d\tau \right\rangle_\omega /S(A) \\
\mathbf{x} &\to g_\omega^\tau(\mathbf{x}) : \ evolution\ under\ \Phi_\omega(\mathbf{x}),\ time\ \tau. \\
\chi_A(\mathbf{x}) &= 1\ if\ x \in A,\ otherwise = 0 \\
T_{\rho,A} &= mean\ sojourn\ fraction\ in\ A. \\
S(A) &= area\ of\ \ A
\end{aligned}
$$

Ergodic theorem

- Uniform distribution of the initial positions.
- Mixed average: over time **t** and the ensemble of the random potential <…>.
- The Hamiltonian form =>von Neumann mean ergodic theorem=>
- Variational principle for computing the trapping fraction.

$$
\begin{aligned}
T_A &= \frac{1}{s} \inf_{g_\omega(x)} J\{g_\omega(.)\} \\
J\{g_\omega\} &= \int_{\mathbf{R}^2} \left\langle \{\chi_A(x) - [g_\omega, \Phi_\omega]\}^2 \right\rangle_\omega d^2\mathbf{x} \\
g_\omega(\mathbf{x}) &= \textit{arbitrary stochastic function} \\
& [g_\omega(\mathbf{x}), \Phi_\omega(\mathbf{x})] \quad \textit{Poisson bracket}
\end{aligned}
$$

Upper bound, trial functions 1

$$
\begin{aligned}
g_\omega(\mathbf{x}) &= \sum_{n=1}^{N} f_n(\mathbf{x}) h_{n,\omega} \\
f_n(\mathbf{x}) &= \textit{arbitrary functions} \\
h_{n,\omega} &= u_n\left\{\Phi_\omega(\mathbf{a}_1), \Phi_\omega(\mathbf{a}_2), ..., \Phi_\omega(\mathbf{a}_M)\right\} \\
\mathbf{a}_i &= \textit{arbitrary points in plane}
\end{aligned}
$$

Trial functions 2

$$
\begin{aligned}
g_\omega(\mathbf{x}) &= \sum_{n=1}^{N} [\Phi_\omega(\mathbf{x}), f_n(\mathbf{x})]\, f_n(\mathbf{x}) h_{n,\omega} \\
f_n(\mathbf{x}) &= \textit{arbitrary functions} \\
h_{n,\omega} &= u_n \{\Phi_\omega(\mathbf{a}_1), \Phi_\omega(\mathbf{a}_2), ..., \Phi_\omega(\mathbf{a}_M)\} \\
\mathbf{a}_i &= \textit{arbitrary points in plane}
\end{aligned}
$$

Numerical method

- Set 2 of trial functions
- The **x** functions vanishes outside the domain A.
- Optimisation with parameter a in the scaling f(**x**)->f(a**x**)
- Domain A with radial symmetry
- In the case of self similar random potential the trapping coefficient is independent of radius.

Result, upper bound