Theoretical modeling of error fields penetration and neoclassical toroidal viscosity non-resonant magnetic braking effects in tokamak plasmas

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-*Magnetic error fields* are small non-axisymmetric deviations from the equilibrium magnetic field structure, due to the inherent, slight errors in the positioning of the coils in a tokamak.

- The error fields are responsible for the solid increase of the perturbed flux amplitudes of the MHD instabilities as their marginal stability is approached.

- The error fields augment the *neoclassical toroidal viscosity* (NTV) destabilizing influence on MHD instabilities.

-The electromagnetic torques that develop at the levels of inner plasma inertial layers (at *rational surfaces)* increase the destabilizing effect of the NTV torques at the corresponding non-ideal MHD layers.

-Consequently, the non-resonant (i.e. coupled) error field increases the NTV influence that brakes the toroidal plasma rotation globally, at the level of every inner plasma rational surface.

-Our task is to *mathematically* describe the above phenomena.

The dynamic cylindrical model in low inverse aspect ratio approximation is already done.

The toroidal angular equation of motion of the inertial layer at the rational surface $r = r_s$ is:

$$\rho \frac{\partial \Omega_{zs}}{\partial t} = -\frac{\eta}{\delta_s^2}\Omega_{zs} + \frac{1}{2\mu_0 R_0 r_s \delta_s}\sum_{j,k} k\,\mathrm{Im}\left(\Delta\Psi_s^{jk}\Psi_s^{jk^*}\right)$$

NTV torque   Electromagnetic torque

Ω_{zs} toroidal angular velocity

To be solved, the above equation requires:

a) all sideband perturbed magnetic fluxes at the inertial layer, $r_{s,}$ to be known, $\Psi^{jk}(r_s)$

b) all sideband perturbed magnetic fluxes radial derivative *jump* across the inertial layer at r_s to be known, $\Delta\Psi^{jk}(r_s)$

More precisely:

a) every mode (Fourier) of instability that develops at a rational surface, contributes to a neighboring inertial layer dynamics that corresponds to another rational surface.

b) mode coupling is crucial in explaining the deceleration of every inertial layer rotation

c) a *resonant* error field acts locally

d) a *non-resonant* error field acts globally, at the level of every rational surface

Immediate task: to find Ψ_s^{jk} from plasma, feedback and jump equations.

We have derived:

The ***dynamic plasma equations:***

$$\sum_{j,k}\Big[s_{jk4}^{mn}\frac{\partial^4}{\partial t^4}-(4ik\Omega_z s_{jk4}^{mn}-s_{jk3}^{mn})\frac{\partial^3}{\partial t^3}-(6k^2\Omega_z^2 s_{jk4}^{mn}+3ik\Omega_z s_{jk3}^{mn}-s_{jk2}^{mn})\frac{\partial^2}{\partial t^2}+(4ik^3\Omega_z^3 s_{jk4}^{mn}-3k^2\Omega_z^2 s_{jk3}^{mn}-2ik\Omega_z s_{jk2}^{mn}+s_{jk1}^{mn})\frac{\partial}{\partial t}$$

$$+k^4\Omega_z^4 s_{jk4}^{mn}+ik^3\Omega_z^3 s_{jk3}^{mn}-k^2\Omega_z^2 s_{jk2}^{mn}-ik\Omega_z s_{jk1}^{mn}+s_{jk0}^{mn}\Big]\Psi_s^{jk}+\sum_{j,k}\left\{s_{jkl}^{mn}\to t_{jkl}^{mn},\quad l=\overline{0,4}\right\}\frac{\partial\Psi_s^{jk}}{\partial t}=0$$

The coefficients s_{jkl}^{mn} and t_{jkl}^{mn} are explicitly derived and describe the intrinsic behavior of the plasma, including the viscous stress tensor influence.

The ***dynamic feedback equations*** :

$$\sum_{j,k}\left(\sum_{i=0}^{2}W_{mni}^{jk}\frac{\partial^i}{\partial t^i}\right)\left[u_{ps}^{jk}\frac{\partial\Psi_s^{jk}}{\partial r}+u_{tt}^{jk}\frac{\partial^2\Psi_s^{jk}}{\partial t^2}+\left(\sum_{i=0}^{2}u_{ti}^{jk}\Omega_z^i\right)\frac{\partial\Psi_s^{jk}}{\partial t}+\left(\sum_{i=0}^{2}u_{si}^{jk}\Omega_z^i\right)\Psi_s^{jk}\right]=\sum_{j,k}\left(\sum_{i=0}^{2}A_{mni}^{jk}\frac{\partial^i}{\partial t^i}\right)\Psi_s^{jk}+\sum_{j,k}E_{mn}^{jk}\Psi_{error}^{jk}$$

- A thin inhomogeneous resistive wall and an active system consisting of a number of rectangular, radially thin coils and detectors centred at the same local coordinates are used.

- The coefficients give all the information about the position, disposal, resistive wall inhomogeneity and amplification amplitude.

Ψ_{error}^{jk} describes the magnetic error field spectrum.

We have used:

The **jump of the perturbed magnetic flux** across the inertial r_s layer, valid in cylindrical geometry (Z. Chang et al., *Phys. Plasmas*, 5 (1998) 1076):

$$\Delta\Psi_s^{jk}=-\sum_{j'}\frac{j'\pi}{r_s\tan(\pi\sigma_s/2)}\left(\sigma_s\delta_{j'j}+\alpha_{j'j}\right)\Psi_s^{j'k}$$

All the above Laplace transformed equations form a complete inhomogeneous algebraic system in the Laplace transformed flux and radial derivative flux perturbations.

Using the Laplace transformation, the partial fraction decomposition and the inverse Laplace transformation, we are finally able to obtain the general expression of the *(j,k)* magnetic flux perturbations at a rational surface r_s, to be inserted into the toroidal angular equation of motion of the inertial layer. ***We have obtained*** the following solution to be checked:

$$\Omega_{zs}(t)=\Omega_{zs0}\exp\left(-\frac{\eta}{\rho\delta_s^2}t\right)+\frac{\beta_s}{2}\sum_{\substack{j,j',k\\ j'\neq j}} jk\ \mathrm{Im}\left\{\alpha_{j'j}\sum_{m,n=1}^{L_0}\frac{\beta_m^{j'k}\beta_n^{jk^*}}{\rho(\tau_m+\tau_n^*)+\eta/\delta_s^2}\left[\exp\left(-\frac{\eta}{\rho\delta_s^2}t\right)-\exp\left(\tau_m+\tau_n^*\right)t\right]\right\}$$

error field mode coupling modifies plasma rotation velocity and changes the NTV dependence

Two dependencies have been drawn: - Fig.1 - for a *(2,1)* inner plasma layer

- Fig.2 - for a *(3,1)* boundary plasma layer

It is clearly shown - the plasma deceleration and rotation braking

- the error field penetration that increases the NTV destabilizing influence

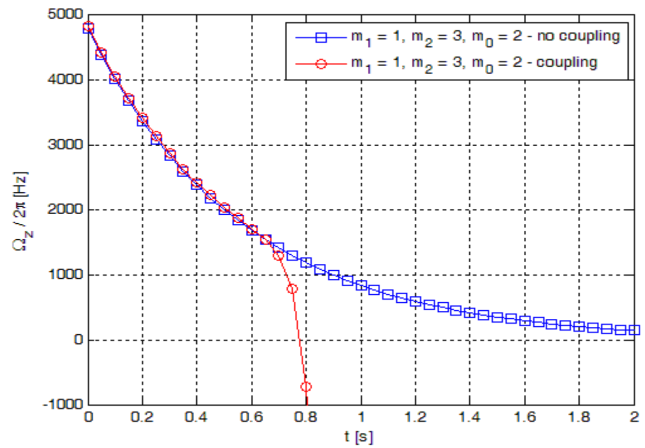

Fig.1

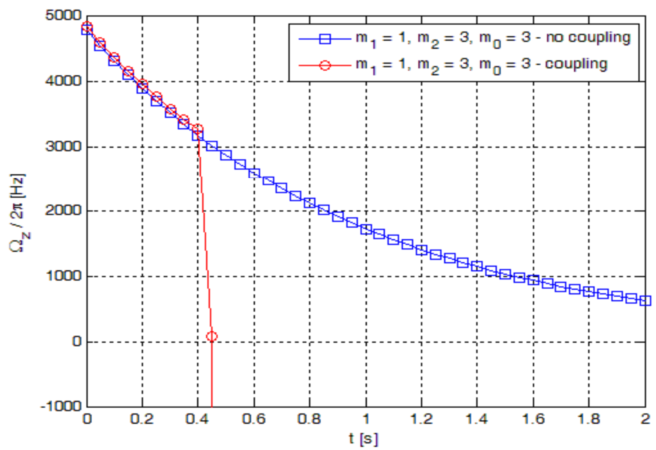

Fig.2

Conclusions:

- The presence of the non-resonant magnetic error field increases the NTV dependence of the toroidal plasma rotation.

- All the spectrum of the non-resonant error field contributes to the toroidal deceleration of a certain plasma inertial layer, but only due to the mode coupling process.

- It seems that, the single mode theory is unable to explain the error field penetration and NTV non-resonant magnetic braking effects phenomena. Only the multimode theory is able.

- The analytically obtained solution makes possible to find the optimal less destabilizing error field spectrum as well as the optimal choice for the feedback parameters in order to provide stability.

The 2010 task to be fulfilled consists in the 2-Dimensional theoretical description of all the above phenomena

The objectives are:

(a) Determination of a general multimode resistive wall modes (RWM) dispersion relation for 2-D axisymmetric geometry.

(b) Derivation of the evolution equations for the plasma angular motion at the level of the plasma boundary and inner non-ideal MHD layers to prove global plasma deceleration and NTV braking of the plasma rotation.

(c) Electromagnetic and NTV torques calculation for shapes of the flux surfaces structure that include toroidicity, ellipticity, triangularity. The influence of the above parameters in finding the optimal less destabilizing error field spectrum will be calculated.

First 2010 objective achieved:

Determination of a general multimode RWM dispersion relation for 2-D axisymmetric geometry

- A 2-dimensional model provides a more realistic description of the phenomena involved by taking into account realistic plasma shape parameters.

-The calculated metric coefficients will be functions of the above mentioned parameters.

- The plasma and feedback equations will be also used within the dynamic description of the NTV braking plasma rotation.

- The RWM dispersion relation is derived within the RWM growth rate description, γ_0
$\partial / \partial t = \gamma_0 + i(m\Omega_\theta - n\Omega_z)$, $\Omega_{\theta,z}$ are the edge plasma poloidal and toroidal angular velocities

- In order to construct an analytic model, the following approximations are considered:
  - *low inverse aspect ratio approximation (the calculus will be performed within the* $\vartheta(\varepsilon^2)$ *approximation, where* $\varepsilon = a / R_0$ *is the inverse aspect ratio);*
  - *thin resistive wall approximation*
  - *the resistive wall and the active feedback coils and detectors are supposed to lie on magnetic surfaces.*

The objective is structured as:

1) Equlibrium description

- The covariant metric tensors, $\{g_{ij}\}_{i,j=1,2,3}$, and the jacobian are calculated, whereas the Shafranov shift, ellipticity and the triangularity satisfy the following equation:

$$r\left(r\Lambda'_j\right)' = \left(\frac{r^2}{a^2} - \frac{2rq^2 p'}{\varepsilon^2}\right)\delta_{|j|1} + \left(j^2 - 1\right)\Lambda_j - 2r(1-s)\Lambda'_j + \vartheta(\varepsilon^2) \quad \text{where} \quad \Lambda_j \equiv \Delta\delta_{|j|1} + E\delta_{|j|2} + T\delta_{|j|3}$$

2) Perturbed MHD equations

We have obtained the following 2-dimensional axisymmetric perturbed plasma system of equations at plasma boundary, within $\vartheta(\varepsilon^2)$ *approximation:*

$$\sum_{j=0}^{4} \gamma_0^j \sum_{l=j}^{4} \binom{l}{j} \sum_{h=m-3}^{m+3} [i(h\Omega_\theta - k\Omega_\varphi)]^{l-j} \left(P^l_{h,n,h-m}\, \phi_a^{h,n} + \widetilde{P}^l_{h,n,h-m}\, {\phi_a^{h,n}}' \right) = 0$$

with the following perturbed magnetic field parameterization: $\mathbf{b} = \nabla \times [(1/B\, \nabla\phi \times \mathbf{B}) \times \mathbf{B}]$

- The above derived coefficients give all the information about plasma parameters.

- The mode coupling is limited to six adjacent neighboring poloidal modes of the central harmonic (three more negative and three more positive poloidal modes), whereas the equilibrium quantities axisymmetry provides no toroidal mode coupling. However, within a higher approximation, more adjacent poloidal modes can be considered.

3) Solving Laplace equation in vacuum in 2-dimensional geometry

Using the ε ordering scheme, we have analytically derived the following solutions:

$$\chi^{m,n}(r) = c_{1r_1r_2}^{m,0}(r)\chi_{r_{1+}}^{m,n} + c_{2r_1r_2}^{m,0}(r)\chi_{r_{2-}}^{m,m} + \sum_{\substack{j=-3 \\ j\neq 0}}^{3}\left[c_{1r_1r_2}^{m,j}(r)\chi_{r_{1+}}^{m+j,n} + c_{2r_1r_2}^{m,j}(r)\chi_{r_{2-}}^{m+j,n}\right] + \vartheta(\varepsilon^2)$$

with the following ε-ordered coefficients $\{c_{ir_1r_2}^{m,0}(r)\}_{i=1,2;\ j=-3,-2,-1,1,2,3} \sim \vartheta(1)$, $\{c_{ir_1r_2}^{m,j}(r)\}_{i=1,2;\ j=-3,-2,-1,1,2,3} \sim \vartheta(\varepsilon)$

The vacuum perturbed magnetic field is parameterized as following: $\mathbf{b} = \nabla\chi$

4) Perturbed feedback equations

Keeping into account the different parameterizations of **b** inside (ϕ) and outside (χ) the plasma, we have finally obtained the system of equations at the plasma boundary:

$$\sum_{j=0}^{2}\gamma_0^j \sum_{h,k}\left(F_{m,n}^{j,h,k}\,\phi_a^{h,k} + \widetilde{F}_{m,n}^{j,h,k}\,\phi_a^{h,k\,\prime}\right) = 0$$

The above derived coefficients give all the information about the feedback parameters.

5) The RWM dispersion relation

The plasma and feedback equations form now a complete algebraic system whose determinant is zero.

Using the Leibniz expansion of determinants and the index ordering transformation [I.G. Miron, *Plasma Phys. Control. Fusion* 50 (2008) 095003], the following RWM dispersion relation is obtained:

$$\sum_{k=0}^{6L} \gamma_0^k \sum_{\substack{l_1,..,l_{2L}=1 \\ distinct}}^{2L} \mathrm{sgn}(l_1,..,l_{2L}) \sum_{\substack{\alpha_1,..,\alpha_{2L}=0 \\ \alpha_1+...+\alpha_{2L}=k}}^{4} \prod_{s=1}^{2L} \Gamma_{s\alpha_s}^{l_s} = 0$$

The coefficients $\Gamma_{s\alpha_s}^{l_s}$ are explicitly derived coefficients as functions of the plasma boundary and feedback parameters.

Conclusions:

- A full theoretical axisymmetric 2-dimensional model for the RWM stabilization has been built.
- A compact polynomial dispersion relation has been obtained.
- No complicated numerical codes are needed. A simple Matlab code to solve a polynomial equation is sufficient to find the RWM growth rate.

The next objectives to be achieved within the EFDA Task Agreement upgrade the cylindrical description to the axisymmetric 2-dimensional description of the error field penetration and NTV non-resonant magnetic braking effects

-The plasma and feedback equations used in the derivation of the 2-D RWM dispersion relation will still be used with the well-thought substitution $\gamma_0 \to \partial/\partial t$

- The crucial aspect to be solved: finding a 2-D expression for the perturbation jump across any plasma inertial layer.

Input data: m_1, m_2, n_1, n_2, ρ_a, η_a, ν, c_{sa}, q_a, q_0, s_a, Δ_a, Δ'_a, E_a, E'_a, T_a, T'_a, r_w, r_f, r_d, R, δ_f, $\delta\theta_f$, $\delta\varphi_f$, $\Delta\theta_f$, $\Delta\varphi_f$, $\Delta\theta_d$, $\Delta\varphi_d$, G_d, G_p, δ_{w1}, δ_{w2}, η_{w1}, η_{w2}, $\Delta\varphi_{w2}$, N, M, $\{\theta_p\}_{p=1,..,M}$.