Extension to qualitative beliefs

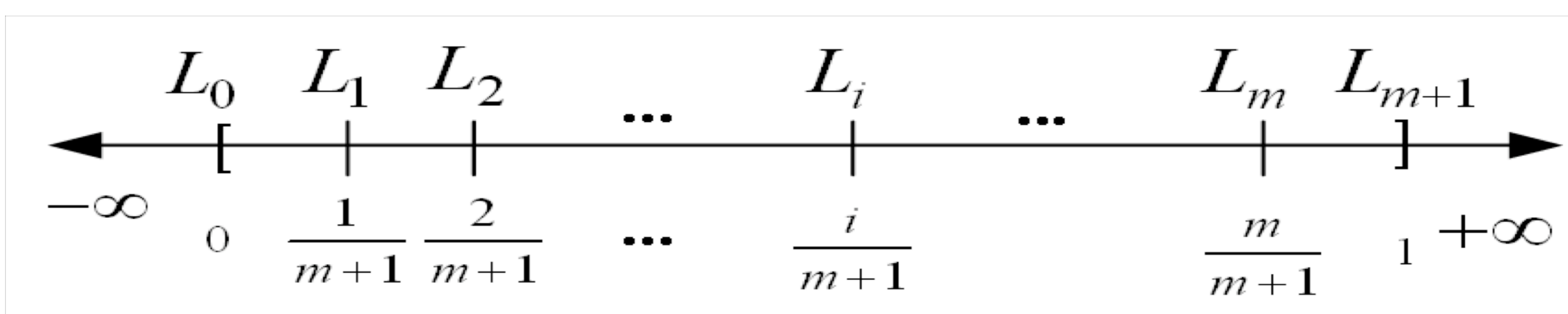
The frame and the DSm models are the same as for quantitative beliefs

The (qualitative) masses/bba's are now defined by linguistic values (labels) taking values in $L = \{L_0, L_1, L_2, \ldots, L_m, L_{m+1}\}$ in such a way that

$$L_0 \prec \underset{\text{very low,}}{L_1} \prec \underset{\text{low,}}{L_2} \prec \ldots \prec L_m \prec L_{m+1}$$

Assumption : We consider linguistic labels as equidistant

Idea : Define operators on labels by mapping L within [0,1], and works also with refined labels to avoid approximations in results.



Basic qualitative operators for labels

- Label addition :

$$L_a + L_b = L_{a+b}$$

since $\frac{a}{m+1} + \frac{b}{m+1} = \frac{a+b}{m+1}$.

- Label multiplication :

$$L_a \times L_b = L_{(ab)/(m+1)}$$

since $\frac{a}{m+1} \cdot \frac{b}{m+1} = \frac{(ab)/(m+1)}{m+1}$.

- Label division (when $L_b \neq L_0$):

$$L_a \div L_b = L_{(a/b)(m+1)}$$

since $\frac{a}{m+1} \div \frac{b}{m+1} = \frac{a}{b} = \frac{(a/b)(m+1)}{m+1}$.

More operators have been defined in the Field and Linear Algebra of Refined Labels (FLARL) [Chap 2, DSmT Book 3].

Qualitative bba: $qm(.) : G^{\Theta} \mapsto L$

Normalized Qualitative bba:

$$qm(\emptyset) = L_0 \triangleq L_{\min} \quad \text{and} \quad \sum_{X \in G^{\Theta}} qm(X) = L_{m+1} \triangleq L_{\max}$$

Fusion of qualitative beliefs

All quantitative fusion rules can be easily exented/adapted for the fusion of qualitative belief thanks to FLARL and qualitative operators for labels.

Linguistic addition & product & masses

Qualitative conjunctive rule:

$$qm_{qCR}(X) = \sum_{\substack{X_1,\ldots,X_s \in 2^{\Theta} \\ X_1 \cap \ldots \cap X_s = X}} \prod_{i=1}^{s} qm_i(X_i)$$

Total qualitative conflict:

$$K_{1\ldots s} = \sum_{\substack{X_1,\ldots,X_s \in 2^{\Theta} \\ X_1 \cap \ldots \cap X_s = \emptyset}} \prod_{i=1}^{s} qm_i(X_i)$$

Qualitative DSmH rule:

$$\phi(X) = L_{m+1} \text{ if } X \notin \emptyset$$
$$\phi(X) = L_0 \text{ otherwise}$$

$$\boxed{\begin{aligned} &qm_{qDSmH}(\emptyset) = L_0 \\ &qm_{qDSmH}(X) \triangleq \phi(X) \cdot \Big[qS_1(X) + qS_2(X) + qS_3(X)\Big] \end{aligned}}$$

$qS_1(X)$, $qS_2(X)$ and $qS_3(X)$ are qualitative counterparts of quantitative functions $S_1(X)$, $S_2(X)$ and $S_3(X)$.

Qualitative PCR5rule:

q-PCR5 has the same expression as classical/quantitative PCR5 formula except that all operators involved in q-PCR5 are qualitative operators on labels defined previously.

Example of qualitative fusion

$\Theta = \{\theta_1, \theta_2\}$ with Shafer's model

$$L = \{L_0, L_1 = \textit{very poor}, L_2 = \textit{poor}, L_3 = \textit{good}, L_4 = \textit{very good}, L_5\} \qquad (m = 4)$$

Normalized inputs

$$qm_1(\theta_1) = L_1, \quad qm_1(\theta_2) = L_3, \quad qm_1(\theta_1 \cup \theta_2) = L_1$$

$$qm_2(\theta_1) = L_2, \quad qm_2(\theta_2) = L_1, \quad qm_2(\theta_1 \cup \theta_2) = L_2$$

Conjunctive consensus

$$\begin{aligned} qm_{12}(\theta_1) &= qm_1(\theta_1)qm_2(\theta_1) + qm_1(\theta_1)qm_2(\theta_1 \cup \theta_2) + qm_1(\theta_1 \cup \theta_2)qm_2(\theta_1) \\ &= L_1 \times L_2 + L_1 \times L_2 + L_1 \times L_2 \\ &= L_{\frac{1\cdot 2}{5}} + L_{\frac{1\cdot 2}{5}} + L_{\frac{1\cdot 2}{5}} = L_{\frac{2}{5}+\frac{2}{5}+\frac{2}{5}} = L_{\frac{6}{5}} = L_{1.2} \end{aligned}$$

Similarly, one will obtain

$$qm_{12}(\theta_2) = qm_1(\theta_2)qm_2(\theta_2) + qm_1(\theta_2)qm_2(\theta_1 \cup \theta_2) + qm_1(\theta_1 \cup \theta_2)qm_2(\theta_2) = L_2$$

$$qm_{12}(\theta_1 \cup \theta_2) = qm_1(\theta_1 \cup \theta_2)qm_2(\theta_1 \cup \theta_2) = L_{0.4}$$

and the conflicting qualitative mass to redistribute

$$qm_{12}(\theta_1 \cap \theta_2) = qm_1(\theta_1)qm_2(\theta_2) + qm_1(\theta_2)qm_2(\theta_1) = L_{1.4}$$

Example of qualitative fusion (cont'd)

Fusion with qDSmH

For the fusion with qDSmH, the mass of $\theta_1 \cap \theta_2$ is transferred to $\theta_1 \cup \theta_2$. Hence:

$$qm_{qDSmH}(\theta_1) = L_{1.2} \qquad qm_{qDSmH}(\theta_2) = L_2 \qquad qm_{qDSmH}(\theta_1 \cap \theta_2) = L_0$$

$$qm_{qDSmH}(\theta_1 \cup \theta_2) = L_{0.4} + L_{1.4} = L_{1.8}$$

Fusion with qPCR5

The conflicting mass $qm_{12}(\theta_1 \cap \theta_2) = L_{1.4}$ is transferred to θ_1 and to θ_2 in the following way:

$$qm_{12}(\theta_1 \cap \theta_2) = qm_1(\theta_1)qm_2(\theta_2) + qm_2(\theta_1)qm_1(\theta_2)$$

Then, $qm_1(\theta_1)qm_2(\theta_2) = L_1 \times L_1 = L_{\frac{1\cdot 1}{5}} = L_{\frac{1}{5}} = L_{0.2}$ is redistributed to θ_1 and θ_2 proportionally with respect to their qualitative masses put in the conflict L_1 and respectively L_1:

$$\frac{x_{\theta_1}}{L_1} = \frac{y_{\theta_2}}{L_1} = \frac{L_{0.2}}{L_1 + L_1} = \frac{L_{0.2}}{L_{1+1}} = \frac{L_{0.2}}{L_2} = L_{\frac{0.2}{2}\cdot 5} = L_{\frac{1}{2}} = L_{0.5}$$

whence $x_{\theta_1} = y_{\theta_2} = L_1 \times L_{0.5} = L_{\frac{1\cdot 0.5}{5}} = L_{\frac{0.5}{5}} = L_{0.1}$

Example of qualitative fusion (cont'd)

Fusion with qPCR5 (cont'd)

$$qm_{12}(\theta_1 \cap \theta_2) = qm_1(\theta_1)qm_2(\theta_2) + qm_2(\theta_1)qm_1(\theta_2) = L_{0.2} + L_{1.2} = \boxed{L_{1.4}}$$

Similarly, $qm_2(\theta_1)qm_1(\theta_2) = L_2 \times L_3 = L_{\frac{2\cdot 3}{5}} = L_{\frac{6}{5}} = L_{1.2}$ has to be redistributed to θ_1 and θ_2 proportionally with L_2 and L_3 respectively :

$$\frac{x'_{\theta_1}}{L_2} = \frac{y'_{\theta_2}}{L_3} = \frac{L_{1.2}}{L_2 + L_3} = \frac{L_{1.2}}{L_{2+3}} = \frac{L_{1.2}}{L_5} = L_{\frac{1.2}{5}\cdot 5} = L_{1.2}$$

whence $\begin{cases} x'_{\theta_1} = L_2 \times L_{1.2} = L_{\frac{2\cdot 1.2}{5}} = L_{\frac{2.4}{5}} = L_{0.48} \\ y'_{\theta_2} = L_3 \times L_{1.2} = L_{\frac{3\cdot 1.2}{5}} = L_{\frac{3.6}{5}} = L_{0.72} \end{cases}$

Adding all these redistibutions to the qualitative masses of θ_1 and θ_2 respectively, one gets:

$$qm_{qPCR5}(\theta_1) = qm_{12}(\theta_1) + x_{\theta_1} + x'_{\theta_1} = L_{1.2} + L_{0.1} + L_{0.48} = L_{1.2+0.1+0.48} = L_{1.78}$$

$$qm_{qPCR5}(\theta_2) = qm_{12}(\theta_2) + y_{\theta_2} + y'_{\theta_2} = L_2 + L_{0.1} + L_{0.72} = L_{2+0.1+0.72} = L_{2.82}$$

$$qm_{qPCR5}(\theta_1 \cup \theta_2) = qm_{12}(\theta_1 \cup \theta_2) = L_{0.4}$$

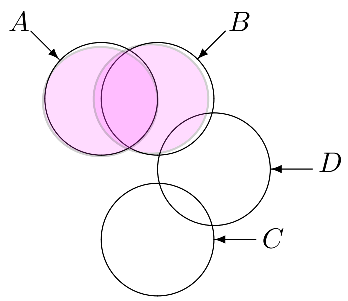
$$qm_{qPCR5}(\theta_1 \cap \theta_2) = L_0$$

which is normalized

Example for qBCR17 (hybrid model)

qBCR17 is the direct extension of BCR17 for qualitative belief based on operators on labels (FLARL). It formula is given in DSmT Book 3.

Example $\Theta = \{A, B, C, D\}$ $L = \{L_0, L_1, L_2, L_3, L_4, L_5, L_6\}$



Input: $qm(A) = L_1, \quad qm(C) = L_1, \quad qm(D) = L_4$

$\text{Conditioning event} = A \cup B$

$qm(D) = L_4$ is transferred in a prudent way to $(A \cup B) \cap D = B \cap D$ according to our hybrid model, because $B \cap D$ is the 1-largest element from $A \cup B$ which is included in D

$qm(C) = L_1$ is transferred to A only, since it is the only element in $A \cup B$ whose qualitative mass $qm(A)$ is different from L_0 (zero).

Result :

$qm_{qBCR17}(A|A \cup B) = L_2$ $\quad qm_{qBCR17}(D|A \cup B) = L_0$

$qm_{qBCR17}(C|A \cup B) = L_0$ $\quad qm_{qBCR17}(B \cap D|A \cup B) = L_4$

Fusion of sources with different importance

The importance of a source is different of its reliability and is specially important in Multicriteria Decision Making (see example in part 4)

Question: How to deal with importance ?

Answer: Use importance discounting with PCR5 (or PCR6) fusion rule

Importance discounting with $\beta \in [0,1]$:

$$\begin{cases} m_\beta(X) = \beta \cdot m(X), & \text{for } X \neq \emptyset \\ m_\beta(\emptyset) = \beta \cdot m(\emptyset) + (1-\beta) \end{cases}$$

$$\neq$$

Reliability discounting with $\alpha \in [0,1]$:

$$\begin{cases} m_\alpha(X) = \alpha \cdot m(X), & \text{for } X \neq \Theta \\ m_\alpha(\Theta) = \alpha \cdot m(\Theta) + (1-\alpha) \end{cases}$$

The importance discounting keeps the specificity of original information contrariwise to classical reliability discounting approach.

The fusion of (importance) discounted bba's is done using

$$m_{PCR5_\emptyset}(X) = \sum_{\substack{X_1,X_2 \in 2^\Theta \\ X_1 \cap X_2 = X}} m_1(X_1)m_2(X_2) + \sum_{\substack{X_2 \in 2^\Theta \\ X_2 \cap X = \emptyset}} \left[\frac{m_1(X)^2 m_2(X_2)}{m_1(X)+m_2(X_2)} + \frac{m_2(X)^2 m_1(X_2)}{m_2(X)+m_1(X_2)}\right]$$

The importance discounting cannot be used efficiently with Dempster's rule since it doesn't respond to the discounting towards the empty set - see [Smarandache-Dezert Fusion 2010]

Example of importance discounting with PCR5 fusion

Example 1: Let's consider $\Theta = \{A, B\}$, Shafer's model, and two sources with respectively bba's $m_1(.)$ and $m_2(.)$ given by $m_1(A) = 0.8$, $m_1(B) = 0.2$ and $m_2(A) = 0.4$, $m_2(B) = 0.6$.

Case 1 : No discounting

	$m_{\beta_1=1}(.)$	$m_{\beta_2=1}(.)$	$m_{12}(.)$	$m_{PCR5}(.)$
$\emptyset$	0	0	0.56	0
A	0.8	0.4	0.32	0.64
B	0.2	0.6	0.12	0.36

Case 2 : Importance discounting with $\beta_1 = 0.2$ $\beta_2 = 0.8$

	$m_{\beta_1=0.2}(.)$	$m_{\beta_2=0.8}(.)$	$m_{12}(.)$	$m^{norm}_{PCR5_\emptyset}(.)$
$\emptyset$	0.80	0.20	0.9296	0
A	0.16	0.32	0.0512	0.43
B	0.04	0.48	0.0192	0.57

Case 3 : Reliability discounting with $\alpha_1 = 0.2$ $\alpha_2 = 0.8$

	$m_{\alpha_1=0.2}(.)$	$m_{\alpha_2=0.8}(.)$	$m_{12}(.)$	$m_{PCR5}(.)$
$\emptyset$	0	0	0.0896	0
A	0.16	0.32	0.3392	0.3698
B	0.04	0.48	0.4112	0.4702
$A \cup B$	0.80	0.20	0.1600	0.1600

Mixing reliability and importance discountings

Method 1: Step 1: Apply reliability discounting, then importance discounting to get $m_{\alpha_i,\beta_i}(.)$. Then use PCR5ø or PCR6ø to combine them;
Step 2: Apply importance discounting, then reliability discounting to get $m_{\beta_i,\alpha_i}(.)$. Then use PCR5ø or PCR6ø to combine them and normalize the bba;
Step 3: Average the results of steps 1 & 2

Method 2: Step 1: Apply reliability discounting to get $m_{\alpha_i}(.)$. Then use PCR5 or PCR6 to combine them;
Step 2: Apply importance discounting, to get $m_{\beta_i}(.)$. Then use PCR5ø or PCR6ø to combine them and then normalize.
Step 3: Average the results of steps 1 & 2

see [Smarandache-Dezert Fusion 2010] paper

Outline

Introduction

Part 1 : Fusion based on belief functions in DST

- Dempster-Shafer Theory (DST)
- Rules of combinations and limitations of DST

Part 2 : Fusion based on belief functions in DSmT

- Dezert-Smarandache Theory (DSmT)
- Modeling, fusion and conditioning for quantitative beliefs
- Extension to qualitative beliefs
- Fusion of sources with different importance

Part 3 : Probabilistic Transformations

Part 4 : Multicriteria Decision Making using DSmT

Part 5 : Browsing some applications

Conclusions & References

Probabilistic transformations

Why ? Useful for decision-making under uncertainty and/or for mixing uncertainty management techniques with purely Bayesian approaches and filtering techniques (i.e. PDAF, JPDAF, MHT, etc)

Purpose: One wants to approximate a bba m(.) by a probabiliy measure P(.)

Solutions: Many solutions exist (Pignistic, Sudano's, Cuzzolin's, DSmP, ...)

Generalized Pignistic Transformation

It allows to build a subjective probability measure P{.} over hyper-power set to help the decision making under uncertainty (other issues are possible).

$$P\{A\} = \sum_{X \in D^\Theta} \frac{\mathcal{C}_\mathcal{M}(X \cap A)}{\mathcal{C}_\mathcal{M}(X)} m(X)$$

$$P\{A\} \equiv \text{betP}\{A\}$$

in Smets' TBM context

$\mathcal{C}_\mathcal{M}(X)$ = DSm cardinality of X = # of parts of X in Venn Diagram for model M under consideration

Classical Pignistic Transform (CPT)

$$P\{A\} = \sum_{X \in 2^\Theta} \frac{|X \cap A|}{|X|} m(X)$$

GPT reduces to CPT when Shafer's model is used

Property : $\text{Bel}(A) \leq P\{A\} \leq \text{Pl}(A)$

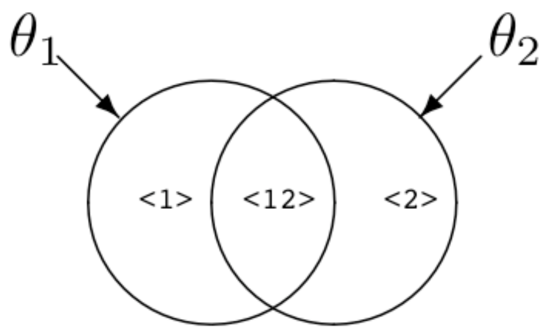
CPT & GPT are not invertible.

Other alternatives exist
[Sudano 2002, 2003, Cuzzolin 2007, DSmP 2008]

Examples for Pignistic Transformation

Based on free-DSm model

$$\Theta = \{\theta_1, \theta_2\} \qquad D^\Theta = \{\emptyset, \theta_1 \cap \theta_2, \theta_1, \theta_2, \theta_1 \cup \theta_2\}$$



$$P\{\emptyset\} = 0$$

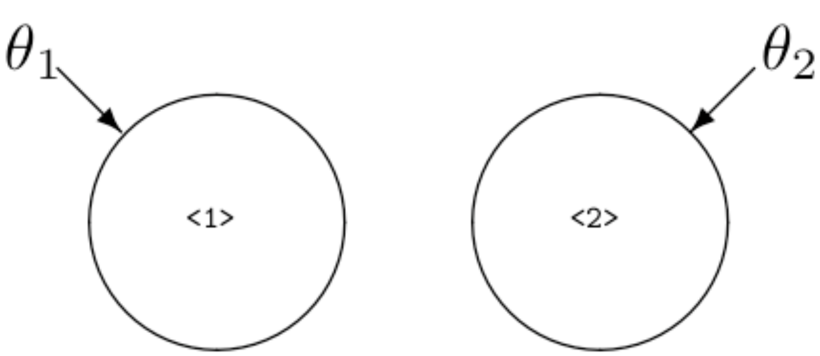
$$P\{\theta_1\} = m(\theta_1) + \frac{1}{2}m(\theta_2) + m(\theta_1 \cap \theta_2) + \frac{2}{3}m(\theta_1 \cup \theta_2)$$

$$P\{\theta_2\} = m(\theta_2) + \frac{1}{2}m(\theta_1) + m(\theta_1 \cap \theta_2) + \frac{2}{3}m(\theta_1 \cup \theta_2)$$

$$P\{\theta_1 \cap \theta_2\} = \frac{1}{2}m(\theta_2) + \frac{1}{2}m(\theta_1) + m(\theta_1 \cap \theta_2) + \frac{1}{3}m(\theta_1 \cup \theta_2)$$

$$P\{\theta_1 \cup \theta_2\} = P\{\Theta\} = m(\theta_1) + m(\theta_2) + m(\theta_1 \cap \theta_2) + m(\theta_1 \cup \theta_2) = 1$$

Based on Shafer's model



$$\theta_1 \cap \theta_2 \overset{\mathcal{M}^0}{\equiv} \emptyset$$

$$P\{\emptyset\} = 0$$

$$P\{\theta_1 \cap \theta_2\} = 0$$

$$P\{\theta_1\} = m(\theta_1) + \frac{1}{2}m(\theta_1 \cup \theta_2)$$

$$P\{\theta_2\} = m(\theta_2) + \frac{1}{2}m(\theta_1 \cup \theta_2)$$

$$P\{\theta_1 \cup \theta_2\} = m(\theta_1) + m(\theta_2) + m(\theta_1 \cup \theta_2) = 1$$

Sudano's Transformations (2001-2006)

All these transformations have been developed in DST framework only.

$$PrPl(X) = Pl(X) \cdot \sum_{Y \in 2^\Theta, X \subseteq Y} \frac{m(Y)}{CS[Pl(Y)]}$$

based on a mapping proportional to plausibility

$$PrBel(X) = Bel(X) \cdot \sum_{Y \in 2^\Theta, X \subseteq Y} \frac{m(Y)}{CS[Bel(Y)]}$$

based on a mapping proportional to credibility

compound-to-sum of singletons (CS) operator

$$CS[f(Y)] \triangleq \sum_{Y_i \in 2^\Theta, |Y_i|=1, \cup_i Y_i = Y} f(Y_i)$$

$$PrNPl(X) = \frac{1}{\Delta} \sum_{Y \in 2^\Theta, Y \cap X \neq \emptyset} m(Y) = \frac{1}{\Delta} \cdot Pl(X)$$

mapping proportional to normalized plausibility

where Δ is a normalization factor.

$$PraPl(X) = Bel(X) + \epsilon \cdot Pl(X)$$

mapping using both Bel(.) and Pl(.)

with $\epsilon \triangleq (1 - \sum_{Y \in 2^\Theta} Bel(Y))/(\sum_{Y \in 2^\Theta} Pl(Y))$

$$PrHyb(X) = PraPl(X) \cdot \sum_{\substack{Y \in 2^\Theta \\ X \subseteq Y}} \frac{m(Y)}{CS[PraPl(Y)]}$$

hybrid mapping

There exist cases where Sudano's transformations cannot be computed (division by zero) and they do not provide n general the max of PIC. PrNPL has an abnormal behavior also:

Say if $\Theta = \{A, B, C\}$ and $m(A) = 0.2$, $m(B) = m(C) = 0$ and $m(B \cup C) = 0.8$, then

$$PrNPl(A) = 0.1112 < m(A) = 0.2$$

Simple example of Sudano's Transformations

$\Theta = \{A, B\}$ with Shafer's model

$G^{\Theta} = 2^{\Theta} = \{\emptyset, A, B, A \cup B\}$

Belief mass

	A	B	$A \cup B$
$m(.)$	0.3	0.1	0.6

$Bel(A) = 0.3$

$Bel(B) = 0.1$

$Pl(A) = 0.9$

$Pl(B) = 0.7$

$PrPl(A) = 0.9 \cdot [0.3/0.9 + 0.6/(0.9 + 0.7)] = 0.6375$

$PrPl(B) = 0.7 \cdot [0.1/0.7 + 0.6/(0.9 + 0.7)] = 0.3625$

$PrBel(A) = 0.3 \cdot [0.3/0.3 + 0.6/(0.3 + 0.1)] = 0.75$

$PrBel(B) = 0.1 \cdot [0.1/0.1 + 0.6/(0.3 + 0.1)] = 0.25$

$PrNPl(A) = 0.9/(0.9 + 0.7) = 0.5625$

$PrNPl(B) = 0.7/(0.9 + 0.7) = 0.4375$

$\epsilon \triangleq (1 - \sum_{Y \in 2^{\Theta}} Bel(Y))/(\sum_{Y \in 2^{\Theta}} Pl(Y))$ $\longrightarrow$ $\epsilon = \frac{1-0.3-0.1}{0.9+0.7} = 0.375$

$PraPl(A) = 0.3 + 0.375 \cdot 0.9 = 0.6375$

$PraPl(B) = 0.1 + 0.375 \cdot 0.7 = 0.3625$

$$PrHyb(A) = 0.6375 \cdot [\frac{0.3}{0.6375} + \frac{0.6}{0.6375 + 0.3625}] = 0.6825$$

$$PrHyb(B) = 0.3625 \cdot [\frac{0.1}{0.3625} + \frac{0.6}{0.6375 + 0.3625}] = 0.3175$$

whereas BetP(.) gives

$BetP(A) = 0.3 + (0.6/2) = 0.60$

$BetP(B) = 0.1 + (0.6/2) = 0.40$

Cuzzolin's Transformation (2007)

Fabio Cuzzolin proposed in 2007 in the DST framework the following transformation based on a geometric interpretation of the Dempster-Shafer combination rule.

Total Non Specific Mass

$$CuzzP(\theta_i) = m(\theta_i) + \frac{\Delta(\theta_i)}{\sum_{j=1}^{n} \Delta(\theta_j)} \times TNSM$$

with $\Delta(\theta_i) \triangleq Pl(\theta_i) - m(\theta_i)$ and $TNSM = 1 - \sum_{j=1}^{n} m(\theta_j) = \sum_{A \in 2^\Theta, |A|>1} m(A)$

Remarks

1 - CuzzP comes from the Cuzzolin's geometric interpretation of DS rule, but DS rule is not very efficient for combining high conflicting belief masses.

2 - CuzzP is not satisfactory since some parts of the masses of partial ignorance, say A, involved in TNSM are redistributed back to singletons which are not included in A.

Example: If $\Theta = \{A, B, C\}$, then $m(A \cup B) > 0$ is redistributed a bit back to C but only A and B are involved in this partial ignorance !!!

3 - CuzzP doesn't not provide max of PIC.

4 - CuzzP is mathematically not defined when m(.) is already a probabilistic mass.

Simple example of Cuzzolin'sTransformation

$\Theta = \{A, B\}$ with Shafer's model

$G^{\Theta} = 2^{\Theta} = \{\emptyset, A, B, A \cup B\}$

Belief mass

	A	B	$A \cup B$
$m(.)$	0.3	0.1	0.6

$Bel(A) = 0.3$

$Bel(B) = 0.1$

$Pl(A) = 0.9$

$Pl(B) = 0.7$

$$TNSM = m(A \cup B) = 0.6$$

$$\Delta(A) = Pl(A) - m(A) = 0.6$$

$$\Delta(B) = Pl(B) - m(B) = 0.6$$

$$CuzzP(A) = m(A) + \frac{\Delta(A)}{\Delta(A) + \Delta(B)} \cdot TNSM = 0.3 + \frac{0.6}{0.6 + 0.6} \cdot 0.6 = 0.60$$

$$CuzzP(B) = m(B) + \frac{\Delta(B)}{\Delta(A) + \Delta(B)} \cdot TNSM = 0.1 + \frac{0.6}{0.6 + 0.6} \cdot 0.6 = 0.40$$

In this particular example, CuzzP(.) coincides with BetP(.)

DSm Probabilistic Transformation (DSmP)

Development of a new probabilistic transformation, denoted DSmP(.), of any basic belief assignment m(.) into a subjective probabilistic measure.

DSmP «maximizes» the Probabilistic Information Content (PIC) of the approximation, contrariwise to other approaches (PIC=1=”Deterministic” distribution, PIC=0=uniform distribution).

Note: DSmP provides the max of PIC WITH numerical robustness of result, but not the max of PIC in absolute value [see Han et al. Fusion 2010]

DSmP(.) is useful for hard or soft decision-making support. It works both in Dempster-Shafer Theory (DST) and Dezert-Smarandache Theory (DSmT).

PIC has been introduced by John Sudano [Fusion 2002]

Probabilistic Information Content (PIC)

Shannon entropy (1948): $$H(P) \triangleq -\sum_{i=1}^{n} P\{\theta_i\} \log_2(P\{\theta_i\})$$

$H(P)$ is maximal for the uniform probability measure. $$H_{\max} = -\sum_{i=1}^{n} \frac{1}{n} \log_2(\frac{1}{n}) = \log_2(n)$$

$H(P)$ is minimal for a "deterministic probability measure". $$H_{\min} = 0$$

PIC = Dual of the normalized Shannon Entropy (Sudano 2002)

$$PIC(P) = 1 + \frac{1}{H_{\max}} \cdot \sum_{i=1}^{n} P\{\theta_i\} \log_2(P\{\theta_i\})$$

$PIC_{\max} = 1$ with any "deterministic probability measure".

$PIC_{\min} = 0$ with the uniform probability measure.

Relationship between PIC(P) and H(P)

$$PIC(P) = 1 - (H(P)/H_{\max})$$

$$H(P) = H_{\max} \cdot (1 - PIC(P))$$

DSmP Formula

DSmP(.) proposes a new proportionalization (using both masses and cardinalities) for transferring the mass of partial ignorances to elements involved into it.

$\mathcal{C}(X \cap Y) =$ DSm Cardinal of $X \cap Y$.

DSmP Formula : $DSmP_{\epsilon}(\emptyset) = 0$

$$\forall X \in G^{\Theta} \setminus \{\emptyset\}$$

$$\boxed{DSmP_{\epsilon}(X) = \sum_{Y \in G^{\Theta}} \frac{\sum_{\substack{Z \subseteq X \cap Y \\ \mathcal{C}(Z)=1}} m(Z) + \epsilon \cdot \mathcal{C}(X \cap Y)}{\sum_{\substack{Z \subseteq Y \\ \mathcal{C}(Z)=1}} m(Z) + \epsilon \cdot \mathcal{C}(Y)} m(Y)}$$

$\epsilon \geq 0$ is a tuning parameter

$\mathcal{C}(Y) =$ DSm cardinal of Y.

Advantages of DSmP: It works directly with any model (free, Shafer's, hybrid)

$\epsilon \to 0$ allows to reach the maximum PIC value of the approximation of $m(.)$ into a proba.

DSmP is actually a generalization of BetP/CPT and of GPT.

Remark: When $\epsilon = 1$ and when the masses of all elements Z having $\mathcal{C}(Z) = 1$ are zero,

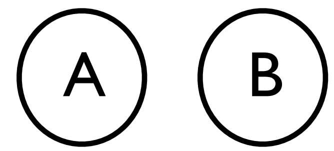
$$DSmP_{\epsilon=1}(.) = BetP(.)$$

All transformations are mathematically idempotent when m(.) is already probabilistic, but Cuzzolin's one.

Simple example for DSmP

$\Theta = \{A, B\}$ with Shafer's model

$\tilde{G}^{\Theta} = 2^{\Theta} = \{\emptyset, A, B, A \cup B\}$



	A	B	$A \cup B$
$m(.)$	0.3	0.1	0.6

$$DSmP_{\epsilon}(A) = \frac{m(A) + \epsilon \cdot \mathcal{C}(A)}{m(A) + \epsilon \cdot \mathcal{C}(A)} \cdot m(A) + \frac{0}{m(B) + \epsilon \cdot \mathcal{C}(B)} \cdot m(B) + \frac{m(A) + \epsilon \cdot \mathcal{C}(A)}{m(A) + m(B) + \epsilon \cdot \mathcal{C}(A \cup B)} \cdot m(A \cup B)$$

$$DSmP_{\epsilon}(B) = \frac{0}{m(A) + \epsilon \cdot \mathcal{C}(A)} \cdot m(A) + \frac{m(B) + \epsilon \cdot \mathcal{C}(B)}{m(B) + \epsilon \cdot \mathcal{C}(B)} \cdot m(B) + \frac{m(B) + \epsilon \cdot \mathcal{C}(B)}{m(A) + m(B) + \epsilon \cdot \mathcal{C}(A \cup B)} \cdot m(A \cup B)$$

$$DSmP_{\epsilon}(A \cup B) = \frac{m(A) + \epsilon \cdot \mathcal{C}(A)}{m(A) + \epsilon \cdot \mathcal{C}(A)} \cdot m(A) + \frac{m(B) + \epsilon \cdot \mathcal{C}(B)}{m(B) + \epsilon \cdot \mathcal{C}(B)} \cdot m(B) + \frac{m(A) + m(B) + \epsilon \cdot \mathcal{C}(A \cup B)}{m(A) + m(B) + \epsilon \cdot \mathcal{C}(A \cup B)} \cdot m(A \cup B)$$

Since we use Shafer's model, $\mathcal{C}(A) = \mathcal{C}(B) = 1$ and $\mathcal{C}(A \cup B) = 2$, the previous expressions reduce to

$$DSmP_{\epsilon}(A) = m(A) + \frac{m(A) + \epsilon}{m(A) + m(B) + 2 \cdot \epsilon} \cdot m(A \cup B)$$

$$DSmP_{\epsilon}(B) = m(B) + \frac{m(B) + \epsilon}{m(A) + m(B) + 2 \cdot \epsilon} \cdot m(A \cup B)$$

$$DSmP_{\epsilon}(A \cup B) = m(A) + m(B) + m(A \cup B) = 1$$

	A	B	$PIC(.)$
$PrNPl(.)$	0.5625	0.4375	0.0113
$BetP(.)$	0.6000	0.4000	0.0291
$CuzzP(.)$	0.6000	0.4000	0.0291
$PrPl(.)$	0.6375	0.3625	0.0553
$PraPl(.)$	0.6375	0.3625	0.0553
$PrHyb(.)$	0.6825	0.3175	0.0984
$DSmP_{\epsilon=0.001}(.)$	0.7492	0.2508	0.1875
$PrBel(.)$	0.7500	0.2500	0.1887
$DSmP_{\epsilon=0}(.)$	0.7500	0.2500	0.1887

min

max

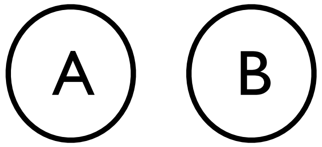
Final results

Results for other 2D examples

Shafer's model and vacuous mass : $m(A \cup B) = 1$

All transformations coincide and give P(A)=P(B)=1/2, but PrBel(.) which is not defined.

Shafer's model with

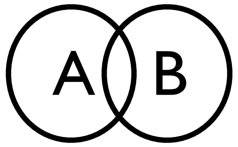


	A	B	$A \cup B$
$m(.)$	0.4	0	0.6

	A	B	$PIC(.)$
$PrBel(.)$	1	**NaN**	**NaN**
$PrNPl(.)$	0.6250	0.3750	0.0455
$BetP(.)$	0.7000	0.3000	0.1187
$CuzzP(.)$	0.7000	0.3000	0.1187
$PrPl(.)$	0.7750	0.2250	0.2308
$PraPl(.)$	0.7750	0.2250	0.2308
$PrHyb(.)$	0.8650	0.1350	0.4291
$DSmP_{\epsilon=0.001}(.)$	0.9985	0.0015	0.9838
$DSmP_{\epsilon=0}(.)$	1	0	1

NaN means "Not a Number"

Free DSm model



	$A \cap B$	A	B	$A \cup B$
$m(.)$	0.4	0.2	0.1	0.3

	A	B	$A \cap B$	$PIC(.)$
$PrNPl(.)$	0.7895	0.7368	0.5263	0.0741
$CuzzP(.)$	0.8400	0.8000	0.6400	0.1801
$BetP(.)$	0.8500	0.8000	0.6500	0.1931
$PraPl(.)$	0.8736	0.8421	0.7157	0.2789
$PrPl(.)$	0.9083	0.8544	0.7627	0.3570
$PrHyb(.)$	0.9471	0.9165	0.8636	0.5544
$DSmP_{\epsilon=0.001}(.)$	0.9990	0.9988	0.9978	0.9842
$PrBel(.)$	**NaN**	**NaN**	1	1
$DSmP_{\epsilon=0}(.)$	1	1	1	1

The refinement of the frame is needed for working with all transformations, but for DSmP formula which can be applied directly.

Results for some 3D examples $\Theta = \{A, B, C\}$

Using Shafer's model

A B C

Mass input

	A	B	C	$A\cup B$	$A\cup C$	$B\cup C$	$A\cup B\cup C$
$m(.)$	0.35	0.25	0.02	0.20	0.07	0.05	0.06

Mass input

	A	B	C	$A\cup B$	$A\cup C$	$B\cup C$	$A\cup B\cup C$
$m(.)$	0.10	0	0.20	0.30	0.10	0	0.30

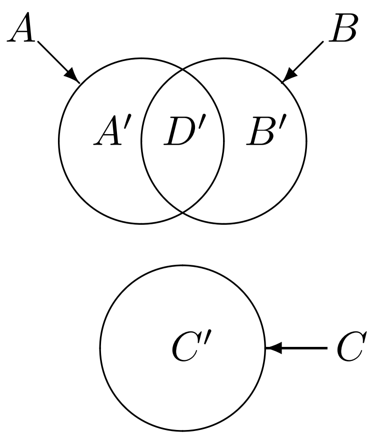
Probabilistic transformations

	A	B	C	$PIC(.)$
$PrNPl(.)$	0.4722	0.3889	0.1389	0.0936
$CuzzP(.)$	0.5029	0.3937	0.1034	0.1377
$BetP(.)$	0.5050	0.3950	0.1000	0.1424
$PraPl(.)$	0.5294	0.3978	0.0728	0.1861
$PrPl(.)$	0.5421	0.4005	0.0574	0.2149
$PrHyb(.)$	0.5575	0.4019	0.0406	0.2517
$DSmP_{\epsilon=0.001}(.)$	0.5665	0.4037	0.0298	0.2783
$PrBel(.)$	0.5668	0.4038	0.0294	0.2793
$DSmP_{\epsilon=0}(.)$	0.5668	0.4038	0.0294	0.2793

	A	B	C	$PIC(.)$
$PrBel(.)$	0.5333	**NaN**	0.4667	**NaN**
$PrNPl(.)$	0.4000	0.3000	0.3000	0.0088
$CuzzP(.)$	0.3880	0.2470	0.3650	0.0163
$BetP(.)$	0.4000	0.2500	0.3500	0.0164
$PraPl(.)$	0.3800	0.2100	0.4100	0.0342
$PrPl(.)$	0.4486	0.2186	0.3328	0.0368
$PrHyb(.)$	0.4553	0.1698	0.3749	0.0650
$DSmP_{\epsilon=0.001}(.)$	0.5305	0.0039	0.4656	0.3500

Results for some 3D examples $\Theta = \{A, B, C\}$

With a hybrid model



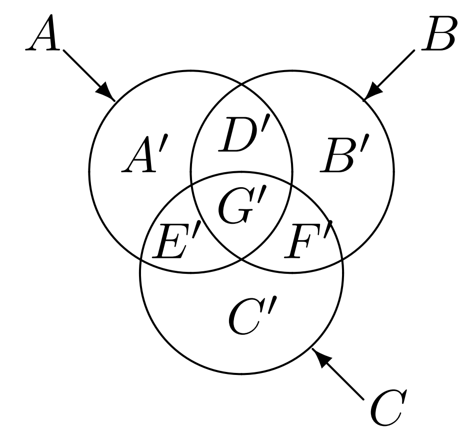
Mass input

	$A \cap B \equiv D'$	$A \equiv A' \cup D'$	$C \equiv C'$
$m(.)$	0.2	0.1	0.2

	$A \cup B \equiv A' \cup B' \cup D'$	$A \cup C \equiv A' \cup C' \cup D'$	$A \cup B \cup C \equiv A' \cup B' \cup C' \cup D'$
$m(.)$	0.3	0.1	0.1

	A'	B'	C'	D'	$PIC(.)$
$PrBel(.)$	**NaN**	**NaN**	0.3000	0.7000	**NaN**
$PrNPl(.)$	0.2728	0.1818	0.1818	0.3636	0.0318
$CuzzP(.)$	0.2000	0.1333	0.2667	0.4000	0.0553
$BetP(.)$	0.2084	0.1250	0.2583	0.4083	0.0607
$PraPl(.)$	0.1636	0.1091	0.3091	0.4182	0.0872
$PrPl(.)$	0.2035	0.0848	0.2404	0.4713	0.1124
$PrHyb(.)$	0.1339	0.0583	0.2656	0.5422	0.1928
$DSmP_{\epsilon=0.001}(.)$	0.0025	0.0017	0.2996	0.6962	0.5390

With the Free model



Mass input

	$A \cap B \cap C$	$A \cap B$	A
$m(.)$	0.1	0.2	0.3

	$A \cup B$	$A \cup B \cup C$	
$m(.)$	0.1	0.3	

Transformations	$PIC(.)$
$PrBel(.)$	**NaN**
$PrNPl(.)$	0.0414
$CuzzP(.)$	0.0621
$PraPl(.)$	0.0693
$BetP(.)$	0.1176
$PrPl(.)$	0.1940
$PrHyb(.)$	0.2375
$DSmP_{\epsilon=0.001}(.)$	0.8986

Qualitative DSmP (qDSmP)

qDSmP Formula :

$$qDSmP_\epsilon(X) = \sum_{Y \in G^\Theta} \frac{\sum\limits_{\substack{Z \subseteq X \cap Y \\ \mathcal{C}(Z)=1}} qm(Z) + \epsilon \cdot \mathcal{C}(X \cap Y)}{\sum\limits_{\substack{Z \subseteq Y \\ \mathcal{C}(Z)=1}} qm(Z) + \epsilon \cdot \mathcal{C}(Y)} qm(Y)$$

Derivation of PIC with "qualitative" probability

Let $\Theta = \{\theta_1, \theta_2, \ldots, \theta_M\}$ and a subjective "qualitative probability" $qP(.)$

i.e. $qP(.) : \Theta \mapsto L = \{L_0, L_1, \ldots, L_n, L_{n+1}\}$, then

$$H(qP) \triangleq -\sum^{M} qP\{\theta_i\} \log_2(qP\{\theta_i\})$$

$$PIC(qP) = 1 + \frac{1}{H_{\max}} \cdot \sum_{i=1}^{M} qP\{\theta_i\} \log_2(qP\{\theta_i\})$$

where all operators involved in qDSmP and PIC formulas are referred to labels (i.e. q-addition, q-multiplication, q-division, etc).

Simple example for qDSmP

$\Theta = \{\theta_1, \theta_2\}$ Shafer model $L = \{L_0, L_1, L_2, L_3, L_4, L_5\}$

Input: $qm(\theta_1) = L_1$, $qm(\theta_2) = L_3$ and $qm(\theta_1 \cup \theta_2) = L_1$

$qm(\theta_1 \cup \theta_2) = L_1$ is redistributed to θ_1 and θ_2 proportionally with respect to their qualitative masses L_1 and L_3 respectively.

Since both L_1 and L_3 are different from L_0, we can take the tuning parameter $\epsilon = 0$ for the best transfer. ϵ is taken different from zero when a mass of a set involved in a partial or total ignorance is zero (for q-masses, it means L_0).

Therefore,

$$\frac{x_{\theta_1}}{L_1} = \frac{x_{\theta_2}}{L_3} = \frac{L_1}{L_1 + L_3} = \frac{L_1}{L_4} = L_{\frac{1}{4} \cdot 5} = L_{\frac{5}{4}} = L_{1.25}$$

and one gets

$$x_{\theta_1} = L_1 \times L_{1.25} = L_{\frac{1 \cdot (1.25)}{5}} = L_{\frac{1.25}{5}} = L_{0.25}$$

$$x_{\theta_2} = L_3 \times L_{1.25} = L_{\frac{3 \cdot (1.25)}{5}} = L_{\frac{3.75}{5}} = L_{0.75}$$

Result

$$qDSmP_{\epsilon=0}(\theta_1 \cap \theta_2) = qDSmP_{\epsilon=0}(\emptyset) = L_0$$

$$qDSmP_{\epsilon=0}(\theta_1) = L_1 + x_{\theta_1} = L_1 + L_{0.25} = L_{1.25}$$

$$qDSmP_{\epsilon=0}(\theta_2) = L_3 + x_{\theta_2} = L_3 + L_{0.75} = L_{3.75}$$

Simple example for qDSmP (cont'd)

Since $H_{\max} = \log_2 n = \log_2 2 = 1$, one obtains:

$$\begin{aligned}
PIC &= 1 + \frac{1}{1} \cdot [qDSmP_{\epsilon=0}(\theta_1) \log_2(qDSmP_{\epsilon=0}(\theta_1)) \\
&\qquad + qDSmP_{\epsilon=0}(\theta_2) \log_2(qDSmP_{\epsilon=0}(\theta_2))] \\
&= 1 + L_{1.25} \log_2(L_{1.25}) + L_{3.75} \log_2(L_{3.75}) \approx L_{0.94}
\end{aligned}$$

since we considered the isomorphic transformation $L_i = i/(m+1)$ (in our particular example $m = 4$ interior labels).

Concluding remarks on DSmP

DSmP provides the maximum of PIC because it is based on proportional redistribution of partial and total uncertainty masses to elements of cardinal one with respect to their corresponding masses and cardinalities.

DSmP works directly for any model (Shafer's, hybrid, or free DSm model) of the frame of the problem and the result can be obtained at any level of precision by the tuning positive parameter $\epsilon > 0$.

$DSmP_{\epsilon=0}$ coincides with Sudano's PrBel transformation for the cases when all masses of singletons involved in ignorances are nonzero.

PrBel formula is restricted to work on Shafer's model only while $DSmP_{\epsilon>0}$ is always defined and for any model.

BetP and Cuzzolin's transformations do not perform well in term of PIC criterion.

DSmP has been extended to the qualitative domain to approximate qualitative belief assignments provided by human sources in natural language into «qualitative» probability.

Outline

Introduction

Part 1 : Fusion based on belief functions in DST

- Dempster-Shafer Theory (DST)
- Rules of combinations and limitations of DST

Part 2 : Fusion based on belief functions in DSmT

- Dezert-Smarandache Theory (DSmT)
- Modeling, fusion and conditioning for quantitative beliefs
- Extension to qualitative beliefs
- Fusion of sources with different importance

Part 3 : Probabilistic Transformations

Part 4 : Multicriteria Decision Making using DSmT

Part 5 : Browsing some applications

Conclusions & References

Multicriteria Decision Making Support using DSmT-AHP

AHP (Analytic Hierarchy Process) is a multi-criteria decision-making method developed by T. Saaty in eighties based on the derivation of priority from preferences.

1) Model the problem as a hierarchy and establish priorities among the elements of the hierarchy by making a series of judgments based on pairwise comparisons matrices. Priority = normalized Perron-Frobenius (PF) vector of the matrix

2) Check the consistency of the judgments and eventually revise the comparison matrices by reasking the experts

3) Synthesize these judgments to yield a set of overall priorities for the hierarchy (weighted arithmetic/ geometric mean).

4) Come to a final decision based on the results (from global priority vector)

Example (Four cars and 3 criteria): $\{Cars\} = \Theta = \{A, B, C, D\}$

$\{\text{Criteria}\} = \{C_1 = \text{Fuel economy}, C_2 = \text{Reliability}, C_3 = \text{Style}\}$

Preferences matrix Ranking criteria:

$$\mathbf{M} = \begin{array}{c} \begin{bmatrix} 1/1 & 1/3 & 4/1 \\ 3/1 & 1/1 & 5/1 \\ 1/4 & 1/5 & 1/1 \end{bmatrix} \\ \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \end{array} \Rightarrow \mathbf{w} = \begin{bmatrix} 0.2797 \\ 0.6267 \\ 0.0936 \end{bmatrix}$$

AHP result

$$[\mathbf{w}(C_1)\ \mathbf{w}(C_2)\ \mathbf{w}(C_3)] = \begin{bmatrix} 0.2500 & 0.4733 & 0.1129 \\ 0.1304 & 0.0611 & 0.4435 \\ 0.5109 & 0.1832 & 0.0565 \\ 0.1087 & 0.2824 & 0.3871 \end{bmatrix}$$

e.g. weight of car 'D' according to criteria C_1

$$\begin{bmatrix} 0.2500 & 0.4733 & 0.1129 \\ 0.1304 & 0.0611 & 0.4435 \\ 0.5109 & 0.1832 & 0.0565 \\ 0.1087 & 0.2824 & 0.3871 \end{bmatrix} \times \begin{bmatrix} 0.2797 \\ 0.6267 \\ 0.0936 \end{bmatrix} = \begin{bmatrix} 0.3771 \\ 0.1163 \\ 0.2630 \\ 0.2436 \end{bmatrix}$$

Weighted Arithmetic Mean

DSmT-AHP is an **extension** of **AHP** using **belief functions** and the **Proportional Conflict Redistribution rule no. 5** proposed in Dezert-Smarandache Theory (**DSmT**) of information fusion.

Main steps of DSmT-AHP:

1) Construction of uncertain comparison matrices and take bba = normalized PF vector of each matrix
2) Use PCR5 rule (or PCR6), to combine bba's to get a final priority ranking.
3) Decision-making (max. of credibility, max. of plausibility, max. of different proba transformations (BetP(.), DSmP(.), etc...)

How to consider different **importances of sources** in the **fusion** ?
Use importance discounting technique on each source combined with non normalized extension of PCR5 or PCR6

Importance discounting with $\beta \in [0,1]$:
$$\begin{cases} m_\beta(X) = \beta \cdot m(X), & \text{for } X \neq \emptyset \\ m_\beta(\emptyset) = \beta \cdot m(\emptyset) + (1-\beta) \end{cases}$$

$$m_{PCR5_\emptyset}(X) = \sum_{\substack{X_1, X_2 \in 2^\Theta \\ X_1 \cap X_2 = X}} m_1(X_1)m_2(X_2) + \sum_{\substack{X_2 \in 2^\Theta \\ X_2 \cap X = \emptyset}} [\frac{m_1(X)^2 m_2(X_2)}{m_1(X)+m_2(X_2)} + \frac{m_2(X)^2 m_1(X_2)}{m_2(X)+m_1(X_2)}]$$

and then normalize the resulting bba.

A simple example for DSmT-AHP (3 cars, 2 criteria)

$$\{Cars\} = \Theta = \{A, B, C\} \qquad \{\text{Criteria}\} = \{C_1 = \text{Fuel economy}, C_2 = \text{Reliability}\}$$

$$\mathbf{M} = \begin{bmatrix} 1/1 & 1/3 \\ 3/1 & 1/1 \end{bmatrix} \approx \begin{bmatrix} 1.0000 & 0.3333 \\ 3.0000 & 1.0000 \end{bmatrix} \Rightarrow \mathbf{w} = \begin{bmatrix} 0.2500 \to \beta_1 \\ 0.7500 \to \beta_2 \end{bmatrix}$$

$$\mathbf{M}(C1) = \left[\begin{array}{c|ccc} & A & B \cup C & \Theta \\ \hline A & 1 & 0 & 1/3 \\ B \cup C & 0 & 1 & 2 \\ \Theta & 3 & 1/2 & 1 \end{array}\right] \Rightarrow \mathbf{w}(C1) \approx \begin{bmatrix} 0.0889 \\ 0.5337 \\ 0.3774 \end{bmatrix}$$

$$\mathbf{M}(C2) = \left[\begin{array}{c|cccc} & A & B & A \cup C & B \cup C \\ \hline A & 1 & 2 & 4 & 3 \\ B & 1/2 & 1 & 1/2 & 1/5 \\ A \cup C & 1/4 & 2 & 1 & 0 \\ B \cup C & 1/3 & 5 & 0 & 1 \end{array}\right] \Rightarrow \mathbf{w}(C2) \approx \begin{bmatrix} 0.5002 \\ 0.1208 \\ 0.1222 \\ 0.2568 \end{bmatrix}$$

$$m_{AHP}(.) = \begin{bmatrix} 0 & 0 \\ 0.0889 & 0.5002 \\ 0 & 0 \\ 0 & 0.1208 \\ 0 & 0 \\ 0 & 0.1222 \\ 0.5337 & 0.2568 \\ 0.3774 & 0 \end{bmatrix} \times \begin{bmatrix} 0.25 \\ 0.75 \end{bmatrix} = \begin{bmatrix} 0 \\ 0.3974 \\ 0 \\ 0.0906 \\ 0 \\ 0.0917 \\ 0.3260 \\ 0.0944 \end{bmatrix}$$

Belief-based AHP solution

Elem. of 2^Θ	$m_{\beta_1,C1}(.)$	$m_{\beta_2,C2}(.)$
$\emptyset$	0.7500	0.2500
A	0.0222	0.3751
B	0	0
$A \cup B$	0	0.0906
C	0	0
$A \cup C$	0	0.0917
$B \cup C$	0.1334	0.1926
$A \cup B \cup C$	0.0944	0

Importance discounting

Elem. of 2^Θ	$m_{C1}(.)$	$m_{C2}(.)$	$m_{PCR5}(.)$
$\emptyset$	0	0	0
A	0.0889	0.5002	0.3837
B	0	0	0.1162
$A \cup B$	0	0.1208	0
C	0	0	0.0652
$A \cup C$	0	0.1222	0.0461
$B \cup C$	0.5337	0.2568	0.3887
$A \cup B \cup C$	0.3774	0	0

PCR5 without importance discounting

Elem. of 2^Θ	$m_{PCR5_\emptyset}(.)$	$m^{normalized}_{PCR5_\emptyset}(.)$
$\emptyset$	0.6558	0
A	0.1794	0.5213
B	0.0121	0.0351
$A \cup B$	0.0159	0.0461
C	0.0122	0.0355
$A \cup C$	0.0161	0.0469
$B \cup C$	0.1020	0.2963
$A \cup B \cup C$	0.0065	0.0188

PCR5 with importance discounting

Elem. of Θ	$Pl(.) - Bel(.)$ with AHP	$Pl(X.) - Bel(.)$ with DSmT-AHP
A	0.2767	0.1118
B	0.5110	0.3612
C	0.5121	0.3619

Outline

Introduction

Part 1 : Fusion based on belief functions in DST

- Dempster-Shafer Theory (DST)
- Rules of combinations and limitations of DST

Part 2 : Fusion based on belief functions in DSmT

- Dezert-Smarandache Theory (DSmT)
- Modeling, fusion and conditioning for quantitative beliefs
- Extension to qualitative beliefs
- Fusion of sources with different importance

Part 3 : Probabilistic Transformations

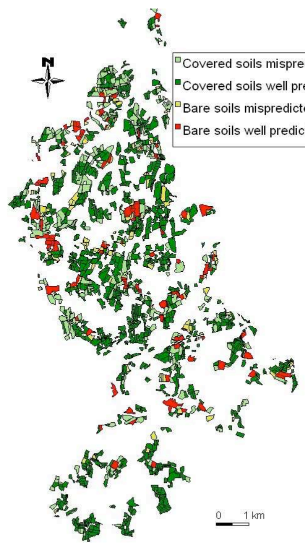
Part 4 : Multicriteria Decision Making using DSmT

Part 5 : Browsing some applications

Conclusions & References

Browsing some applications

[References available in DSmT Books 1,2 & 3]

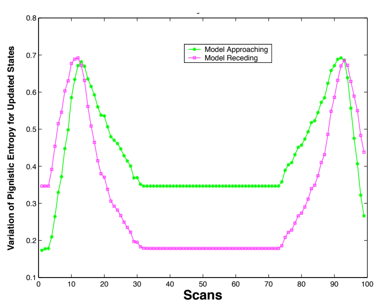


Land cover change prediction for pollution prevention
[Corgne et al. 2004]

SPOT + IRS-LISS III images + GIS + agricultural experts

Power and resource aware distributed smart fusion **[Kadambe 2004]**

Optimization of disparate DSN architecture to minimize power consumption and optimize multitarget detection and classification (typical application of dynamic fusion).



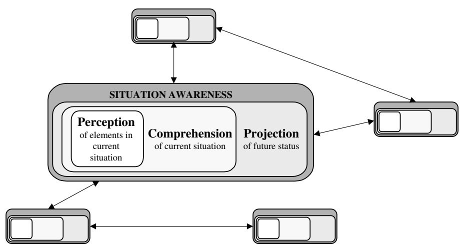
Estimation of target behavior tendencies **[Tchamova et. al 2003]**

Sonar amplitude measurements + fuzzification interface + DSmT

Generalized data association for MTT in clutter **[Tchamova et. al 2004-2006]**

Multitarget tracking with kinematics and attribute measurements

Browsing some applications



Neutrosophic frameworks for situation analysis [Jousselme/Maupin 2004]

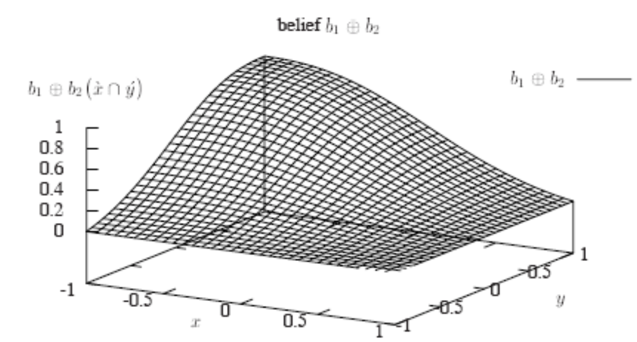
Evaluation of neutrosophic logic and DSmT to cope with the ontological and epistemic problems of situation analysis and awareness

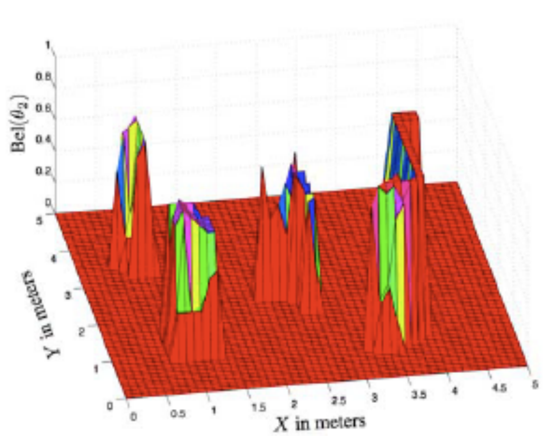
Default reasoning: Solution to the Tweety Penguin Triangle Problem

[Dezert/Smarandache 2004]

Analysis and comparison of Bayesian, Shaferian reasonings w.r.t. DSmT within weighted contradicting rules-based systems.

Fusion with continuous bba's in DSmT [Dambreville 2005-2006]



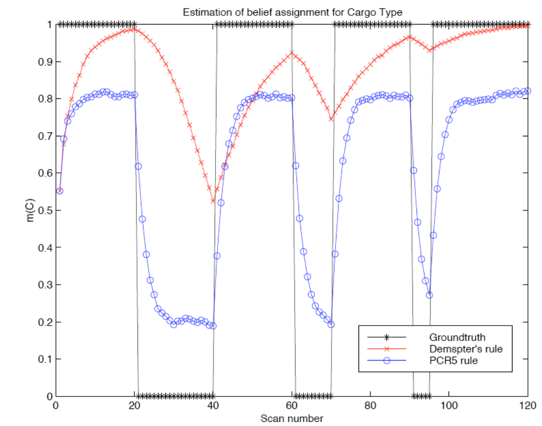


Robot Map building from Sonar Sensors and DSmT (SLAM application)

[Li, Dezert et al. 2006]

Browsing some applications

Target Type Tracking [Dezert, Tchamova et al. 2006]



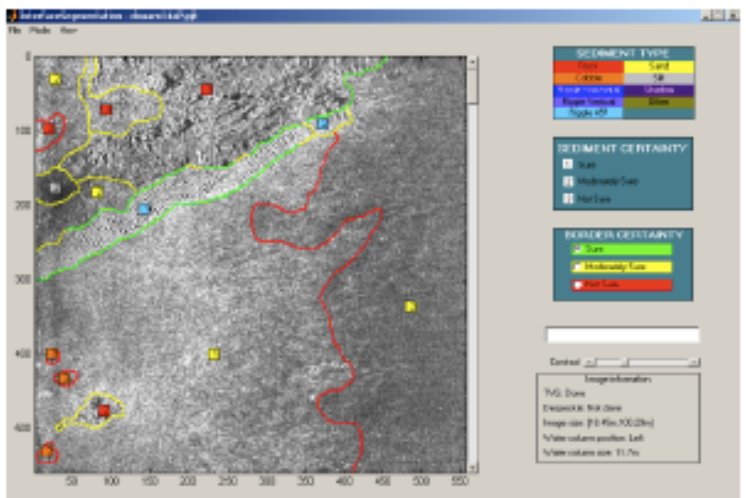
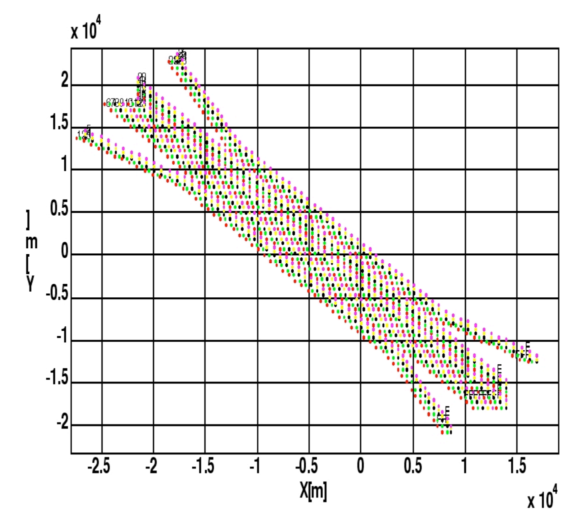


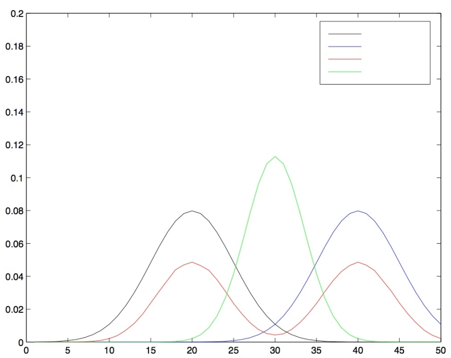
Image segmentation and target classification based on real radar data and PCR rules [Martin, Osswald 2006]

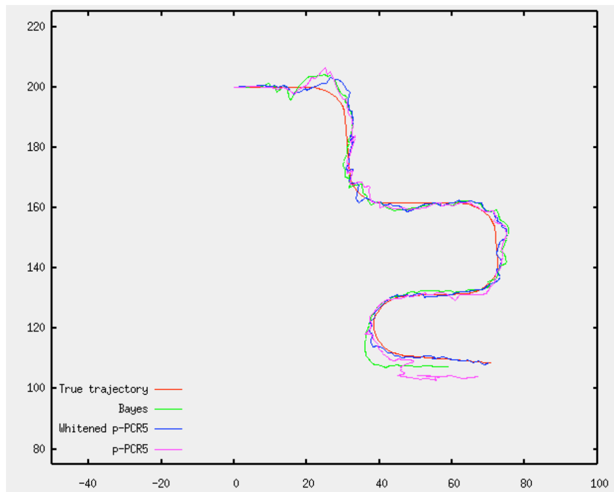


Performance improvement of Multitarget Tracking using DSmT [Tchamova et al. 2005-2006]

Development of a DSmT Matlab toolbox [Djiknavorian & Grenier 2006]
+ PCR algorithms [Martin & Osswald 2006, 2008]

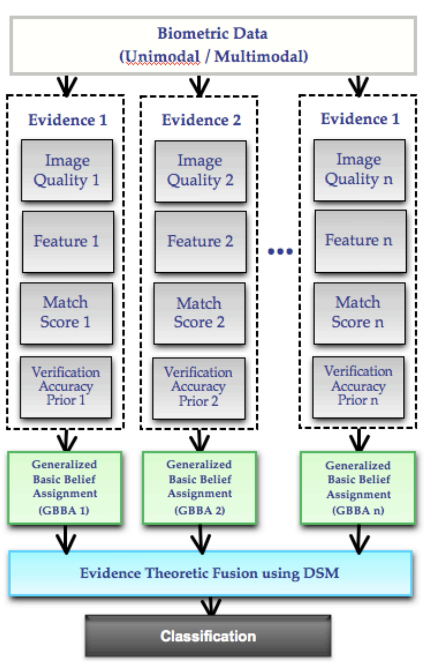
Browsing some applications





MS Particle filtering with PCR5 for target tracking [Kirchner & al. 2007]

Robot Map building and self Localization on real sonar data based on PCR5 [Li & al. 2007]



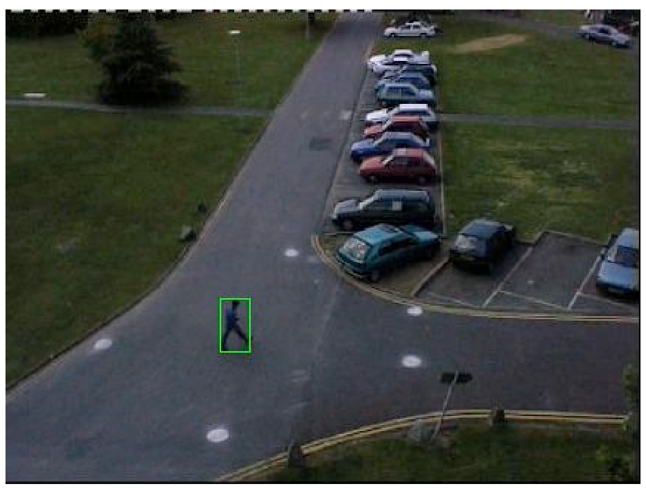
Biometric match score fusion based on DSmT [Vatsa 2008]

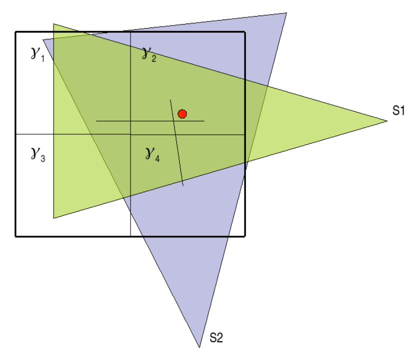
Gallery Images　　Probe Images

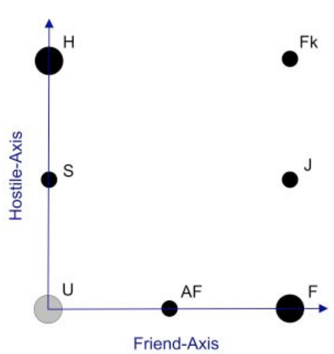
Example of conflicting data – Face recognition algorithm *accepts* and fingerprint recognition algorithm *rejects*

Browsing some applications

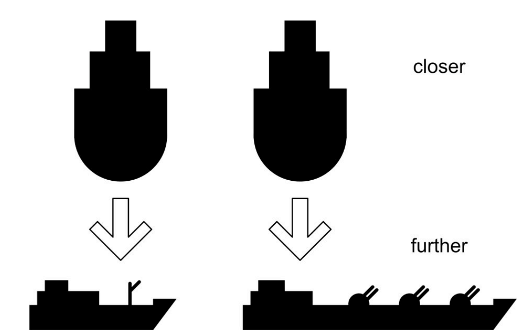
Decision Level Multiple Cameras Fusion Using Dezert-Smarandache Theory, [Garcia, Altamirano 2009]







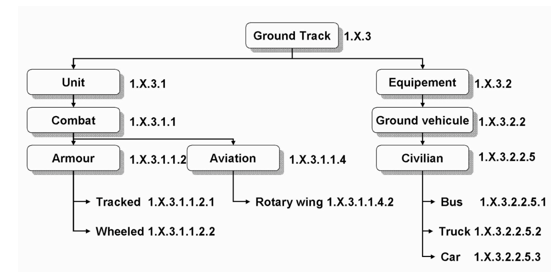
Attribute information evaluation in C&C systems, [Krenc & Kawalec 2009]

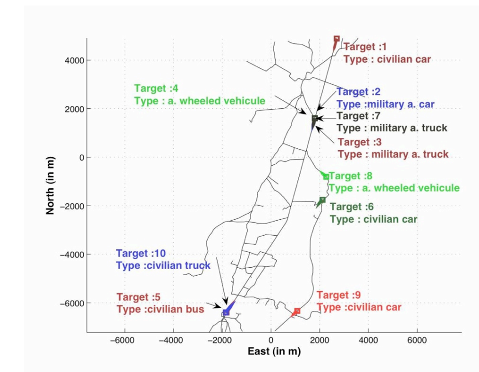


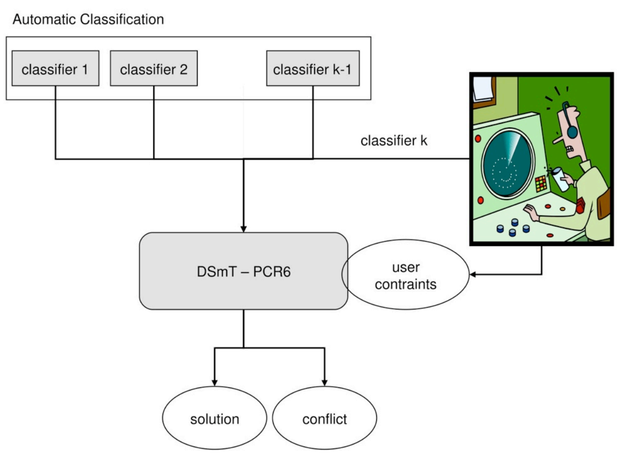
Performance evaluation of tracking algorithms including attribute data [Dezert,Tchamova,Bojilov 2009]

Browsing some applications

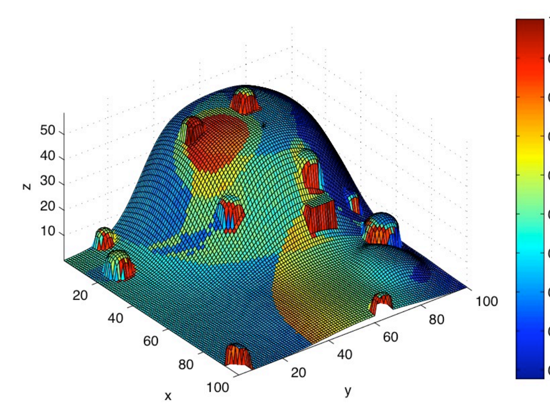
Improvement of multiple ground targets tracking with fusion of identification attributes [Pannetier et al. 2008-2009]



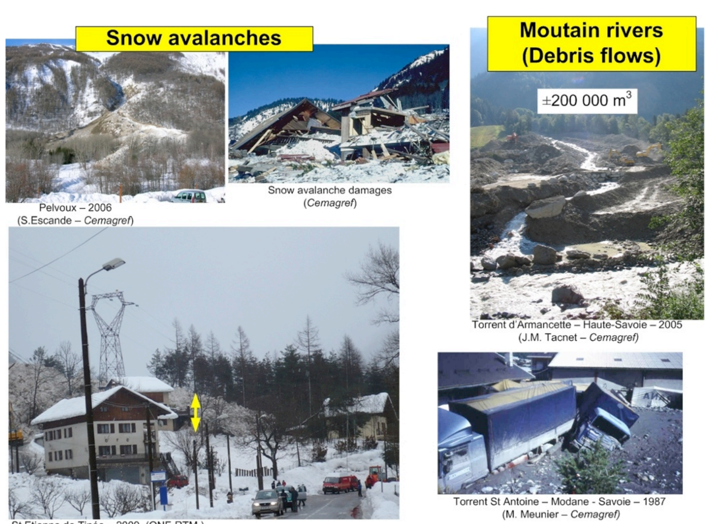




Utilizing classifier conflict for sensor management and user interaction [Van Norden, Jonker 2009]

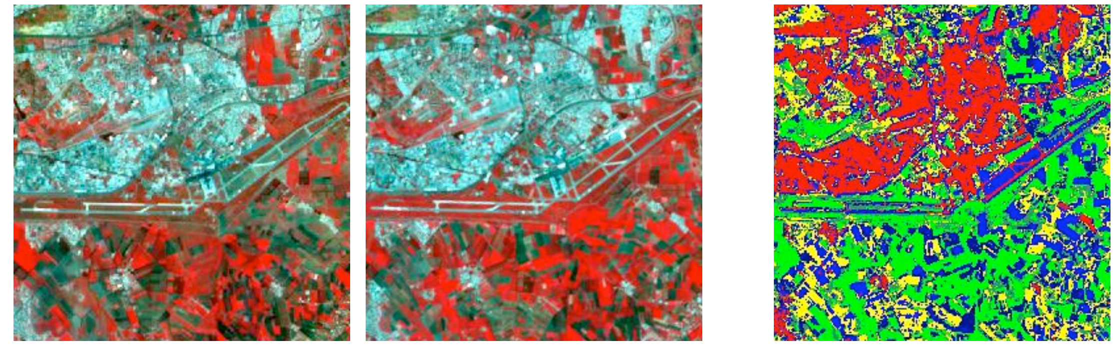


Automatic goal allocation for a planetary rover with DSmT [Vasile,Ceriotti 2009]

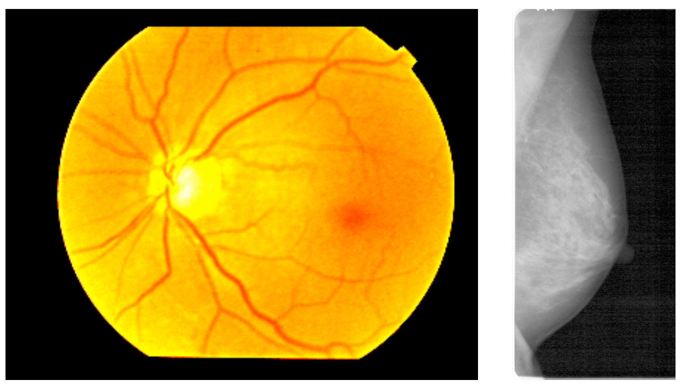


Information fusion for natural hazards in mountains [Tacnet,Batton,Dezert 2009]

Browsing some applications

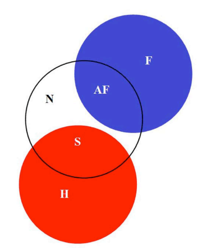


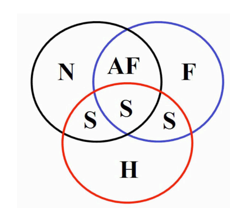
Satellite image fusion using DSmT
[Bouakache,Belhadj-Aissa,Mercier 2009]

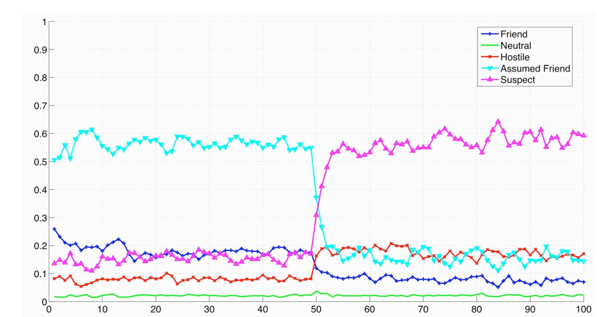


Multimodal information retrieval based on DSmT. Application to computer-aided medical diagnosis [Quellec et al. 2008-2009]

Fusion of ESM allegiance reports using DSmT
[Djiknavorian,Valin, Grenier 2009]







Map regenerating forest stands based on DST and DSmT combination rules
[Mora,Fournier,Foucher 2009]

Processing of information in C2 systems [Krenc 2010]

Maritime surveillance and threat assessment [Van Norden 2010]

Risk prevention against natural hazards in mountains [Tacnet 2009]

Outline

Introduction

Part 1 : Fusion based on belief functions in DST

Dempster-Shafer Theory (DST)
Rules of combinations and limitations of DST

Part 2 : Fusion based on belief functions in DSmT

Dezert-Smarandache Theory (DSmT)
Modeling, fusion and conditioning for quantitative beliefs
Extension to qualitative beliefs
Fusion of sources with different importance

Part 3 : Probabilistic Transformations

Part 4 : Multicriteria Decision Making using DSmT

Part 5 : Browsing some applications

Conclusions & References

Conclusions

DSmT proposes new mathematical foundations for information fusion expressed in terms of quantitative or qualitative beliefs with the following specificities:

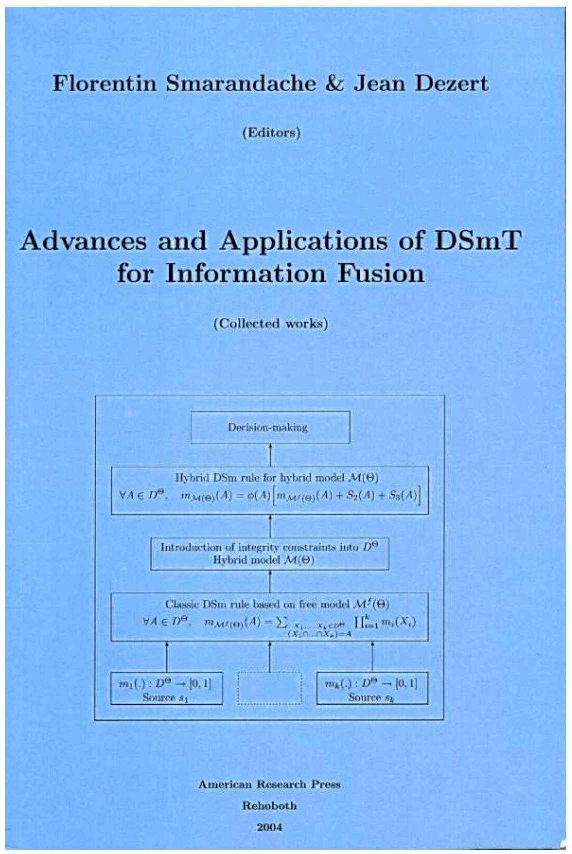
- **DSmT works beyond the limits of the applicability of the DST**
- **DSmT takes into account the intrinsic nature/granularity of information**
- **DSmT works for any models and for static and dynamic fusion**
- **DSmT allows to combine uncertain, conflicting and imprecise quantitative beliefs**
- **DSmT allows to combine uncertain and conflicting qualitative beliefs**
- **DSmT proposes new belief conditioning rules**
- **The reliability and importance of sources, when known, can be easily taken into account**
- **DSmT is a natural extension of previous works done by Yager, Dubois & Prade, Smets and others to circumvent limitations of Dempster's rule through new rules of combination.**

DSmT can be applied in most fusion applications where DST "works on the razor edge", but it can also cover a wider class of applications because of its new appealing specificities. It can also be used for Multicriteria decision making support.

Main References

Advances and Applications of DSmT for Information Fusion, American Research Press, F. Smarandache, J. Dezert Editors

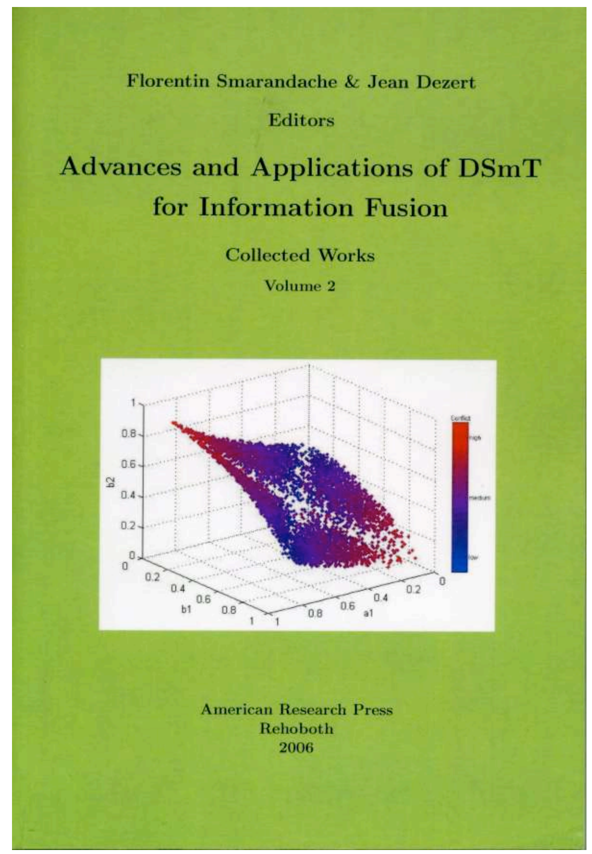
July 2004 - Vol.1



Chinese translation under progress

418 Pages
18 Chapters
17 Contributors

July 2006 - Vol.2



442 Pages
15 Chapters
15 Contributors

June 2009 - Vol.3

758 Pages
25 Chapters
41 Contributors

Freely downloadable in pdf (with other papers) from : http://www.gallup.unm.edu/~smarandache/DSmT.htm

Some additional references

More at: http://www.gallup.unm.edu/~smarandache/DSmT.htm

Generic implementation of fusion rules based on Referee function, Dambreville F. , in Proc. of Workshop on the theory of belief functions, April 1-2, 2010, Brest, France

Non Bayesian Conditioning and Deconditioning, Dezert J., Smarandache F., in Proc of Workshop on the theory of belief functions, April 1-2, 2010, Brest, France.

Multi-criteria decision making based on DSmT-AHP, Dezert J., Tacnet J.-M., Batton-Hubert M., Smarandache F., in Proc. of Workshop on the theory of belief functions, April 1-2, 2010, Brest, France.

A combination of ontology fusion and DSmT as the way to improve the performance of evidential inference in C2 systems, Krenc K., Kawalec A., in Proc. of Workshop on the theory of belief functions, April 1-2, 2010, Brest, France

Extension of Inagaki General Weighted Operators and A New Fusion Rule Class of Proportional Redistribution of Intersection Masses, by F. Smarandache, International Journal of Artificial Intelligence, Vol. 3, No. A09, 79-85, 2009

A New Probabilistic Transformation of Belief Mass Assignment (DSmP), by J. Dezert & F. Smarandache (co-chairs of the special session “Advances and Applications of DSmT for Information Fusion”), Fusion 2008 International Conference, June 30-July 3, 2008, Cologne, Germany.

Application of Probabilistic PCR5 Fusion Rule for Multisensor Target Tracking, by A. Kirchner, F. Dambreville, F. Celeste, J. Dezert, F. Smarandache, Fusion 2007 Int. Conf. on Information Fusion, Québec City, July 2007.

Qualitative Belief Conditioning Rules (QBCR), by F. Smarandache, J. Dezert, Fusion 2007 Int. Conf. on Information Fusion, Québec City, July 2007.

A New Class of Fusion Rules Based on T-Norm and T-Conorm Fuzzy Operators, by A. Tchamova, J. Dezert, F. Smarandache, in "Information & Security / An International Journal", pp. 65-82, Vol. 20, 2006.

Target Type Tracking with PCR5 and Dempster's Rules: A Comparative Analysis, by Jean Dezert, Albena Tchamova, Florentin Smarandache, Pavlina Konstantinova. Fusion 2006 Int. Conf. on Information Fusion, Firenze, Italy, July 2006.

Adaptive Combination Rule and Proportional Conflict Redistribution Rule for Information Fusion, by M. C. Florea, J. Dezert, P. Valin, F. Smarandache, A.-L. Jousselme, Cogis '06 Conference, Paris, March 2006.

General combination rules for qualitative and quantitative beliefs, by A. Martin, C. Osswald, J. Dezert & F. Smarandache, To appear in Journal of Advances in Information Fusion (JAIF), 2009.

Special issue on Fusing Uncertain, Imprecise and Conflicting Information, in Information & Security International Journal, Vol. 20, 2006.

Theses related with DSmT

2010 Ph.D. Thesis, Processing of information in C2 systems with application of Dezert-Smarandache Theory, (in Polish), Military University of Technology, Poland, June 10th, 2010.
2010 Ph.D. Thesis, Sensing, what matters, W. Van Norden (CAMS -- Force Vision), Delft Techn. Univ, Feb 16th, 2010.
2009 Ph.D. Thesis, Prise en compte de l'incertitude dans l'expertise des risques naturels en montagne par analyse multicritères et fusion d'information, J.-M. Tacnet (CEMAGREF), Ecole des Mines de St Etienne, France, Nov. 2009.
2009 Ph.D. Thesis, Cartographie de paramètres forestiers par fusion évidentielle de données géospatiales multi-sources: Application aux peuplements forestiers en regénération et feuillus matures du sud du Québec, Brice Mora, Sherbrooke Univ., Canada, March 5th, 2009.

- 2008 Ph.D. Thesis, Quality Induced Secure Multiclassifier Fingerprint Verification using Extended Feature Set, by Mayank Vatsa, Computer Sciences Dept., West Virginia University, Morgantown, West Virginia, USA, Nov. 2008.
- 2008 Ph.D. Thesis, Indexation et fusion multimodale pour la recherche d'information par le contenu. Application aux bases de données d'images médicales, by G. Quellec, Ecole Nationale Supérieure des Telecommunications de Bretagne, Brest,France, September 19th, 2008.
- 2008 M.Sc. Thesis, Fusion d'informations dans un cadre de raisonnement de Dezert-Smarandache appliquée sur des rapports de capteurs ESM sous le STANAG 1241, by Pascal Djiknavorian, Laboratoire de Radiocommunication et de Traitement du Signal, Laval University, Canada, September 19, 2008.
- 2007 Ph.D. Thesis : Combinaison d'informations hétérogènes dans le cadre unificateur des ensembles aléatoires : Approximations et robustesse, by Mihai Cristian Florea, Université Laval, Québec City, Canada, 13 July 2007.
- 2007 Ph.D. Thesis, Research on Fusion Method of Imperfect Information from Multi-source and Its Application, by Xinde Li,Huazhong University of Science and Technology,Wuhan 430074, P. R. China, June, 2007.
- 2006 M.Sc. Thesis, Designing a Home Security System Using Sensor Data Fusion, by Arezou Moussavi, Islamic Azad University, Southern Unit, Tehran, Iran, September 2006.
- 2004 Ph.D. Thesis, Modélisation prédictive de l'occupation des sols en contexte agricole intensif : application à la couverture hivernale des sols en Bretagne, by Samuel Corgne, Advisor: Laurence Hubert-Moy, Université de Rennes 2, France, 10 December 2004.

Theses under progress

- in defense, by P. Djiknavorian, EE Department, Laval University, Québec, QC, Canada.
- in remote sensing, by Abdenour~Bouakache, Image Processing and Radiation Laboratory,University of Science and Technology Houari Boumediene, Algiers, Algeria.

About Jean Dezert

Jean Dezert was born in l'Hay les Roses, France, on August 25th, 1962. He received the electrical engineering degree from the Ecole Française de Radioélectricité Electronique and Informatique (EFREI), Paris, in 1985, the D.E.A. degree in 1986 from the University Paris VII (Jussieu), and his Ph.D from the University Paris XI, Orsay, in 1990, all in Automatic Control and Signal Processing.

During 1986-1990 he was with the Systems Department at the French Arerospace Lab (ONERA), Châtillon, France, and did research in multisensor multitarget tracking (MS-MTT). During 1991-1992, he visited the Department of Electrical and Systems Engineering, University of Connecticut, Storrs, U.S.A. as an European Space Agency (ESA) Postdoctoral Research Fellow. During 1992-1993 he was teaching assistant in Electrical Engineering at the University of Orléans, France. Since 1993, he is senior research scientist in the Information Modeling and Processing Department (DTIM) at ONERA.

His current research interests include autonomous navigation, multisensor-multitarget tracking (MS-MTT), information fusion, plausible reasoning and non-standard logics. Dr. Jean Dezert ihas developed in collaboration with Prof. Smarandache a new theory of plausible and paradoxical reasoning for information fusion (DSmT) and has edited three textbooks (collected works) devoted to this new emerging research field published by American Research Press, Rehoboth in 2004, 2006 and 2009 respectively. He owns one international patent for autonomous missile navigation and has published several papers in international conferences and journals. He coauthored a chapter in Multitarget-Multisensor Tracking: Applications and Advances, Vol.2 (Y. Bar-Shalom Editor). He was IEEE and Eta Kappa Nu member, taught courses on MS-MTT and Data Fusion at the French ENSTA Engineering School, and serves as reviewer for different International Journals.

He collaborates for the development of the International Society of Information Fusion (ISIF) since 1998, and has served as Local Arrangements Organizer for Fusion 2000 Conf. in Paris. He has been involved in the Technical Program Committees of Fusion 2001-2009 International Conferences. Since 2001, he is a member of the board of the International Society of Information Fusion (http://www.isif.org) and serves in ISIF executive board. He served as executive vice-president of ISIF in 2004. He is also Associate Editor of Journal of Advances in Information Fusion (JAIF).

Address: ONERA/DTIM/SIF, 29 Avenue de la Division Leclerc, 92320 Châtillon, France

Email: jean.dezert@onera.fr

About Florentin Smarandache

Florentin Smarandache was born in Balcesti, Romania, in 1954. He got a M. Sc. Degree in both Mathematics and Computer Science from the University of Craiova in 1979, received a Ph.D. in Mathematics from the State University of Kishinev in 1997, and continued postdoctoral studies at various American Universities (New Mexico State University in Las Cruces, Los Alamos National Laboratory) after emigration. In 1988 he escaped from his country, pasted two years in a political refugee camp in Turkey, and in 1990 emigrated to USA. In 1996 he became an American citizen. Dr. Smarandache worked as a professor of mathematics for many years in Romania, Morocco, and United States, and between 1990-1995 as a software engineer for Honeywell, Inc., in Phoenix, Arizona. In present, he teaches mathematics at the University of New Mexico, Gallup Campus. Very prolific, he is the author, co-author, and editor of 75 books, over 100 scientific notes and articles, and contributed to about 50 scientific and 100 literary journals from around the world (in mathematics, informatics, physics, philosophy, rebus, literature, and arts). He wrote in Romanian, French, and English. Some of his work was translated into Spanish, German, Portuguese, Italian, Dutch, Arabic, Esperanto, Swedish, Farsi, Arabic, Chinese. He was so attracted by contradictions that, in 1980s, he set up the "Paradoxism" avant-garde movement in literature, philosophy, art, even science, which made many advocates in the world, and it's based on excessive use of antitheses, antinomies, paradoxes in creation - making an interesting connection between mathematics, engineering, philosophy, and literature and led him to coining the neutrosophic logic, a logic generalizing the intuitionistic fuzzy logic that is able to deal with paradoxes. In mathematics there are several entries named Smarandache Functions, Sequences, Constants, and especially Paradoxes in international journals and encyclopedias. He organized the 'First International Conference on Neutrosophics' at the University of New Mexico, 1-3 December 2001. Small contributions he had in physics and psychology too. Much of his work is held in "The Florentin Smarandache Papers" Special Collections at the Arizona State University, Tempe, and Texas State University, Austin (USA), also in the National Archives (Rm. Vâlcea) and Romanian Literary Museum (Bucharest), and in the Musée de Bergerac (France). In 2003, he organized with Dr. Jean Dezert, the first special session devoted to plausible and paradoxical reasoning for information fusion at the Fusion 2003 in Cairns, Australia and has participated to several international workshop and seminar on Information Fusion since 2003. He published two books devoted to DSmT-based Information Fusion in 2004 and 2006 respectively. The complete list of references and past seminars and workshop on DSmT can be found on DSmT web page for convenience.

Address: Math. Dept., University of New Mexico, 200 College Road, Gallup, NM 87301, U.S.A.

Email: smarand@unm.edu